

The evolution of speed, size and shape in modern athletics

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SUMMARY

In the present study, we show that the fastest runners and swimmers are becoming not only faster but also heavier, taller and more slender. During the past century, the world record speeds for 100m-freestyle and 100m-dash have increased with body mass (M) raised to the power $1/6$, in accordance with the constructal scaling of animal locomotion. The world records also show that the speeds have increased in proportion with body heights (H) raised to the power $1/2$, in accordance with animal locomotion scaling. If the athlete's body is modeled with two length scales (H , body width L), the (M , H) data can be used to calculate the slenderness of the body, H/L . The world records show that the body slenderness is increasing very slowly over time.

Key words: running, swimming, constructal, animal locomotion, body size scaling, athletics, speed records, world records.

INTRODUCTION

Animal locomotion scaling

Broadly speaking, larger animals move faster on earth than smaller animals (Bejan, 2000; Hoppeler and Weibel, 2005; Weibel, 2000). This aspect of animal scaling is often overlooked because large animals appear sluggish. Geese flap their wings infrequently. Whales swing their tails as if not in a hurry. The scaling relations of animal locomotion have been measured and studied empirically as three separate locomotion mechanisms: flying (Bartholomew and Casey, 1978; Greenewalt, 1975; Lighthill, 1974; Marden et al., 1997; May, 1995; Tennekes, 1997; Wakeling, 1997), running (Heglund et al., 1974; Iriarte-Diaz, 2002; Marsh, 1988; Pennycuik, 1975) and swimming (Arnott et al., 1998; Brett, 1965; Childress and Dudley, 2004; Drucker and Jensen, 1996; Kiceniuk and Jones, 1977; Peake and Farrell, 2004; Rohr and Fish, 2004; Videler, 1993).

More recently, a broader view of the commonality of animal locomotion has been emerging (Ahlborn, 2004; Bejan, 2000; Bejan, 2005; Bejan and Marden, 2006; Bejan and Lorente, 2008; Hoppeler and Weibel, 2005; Marden and Allen, 2002; Muller and van Leeuwen, 2004; Taylor et al., 2003; Weibel, 2000). For example, according to constructal theory, animal locomotion is a rhythm of body motion constructed such that the animal achieves a balance between two expenditures of useful energy: lifting weight on the vertical, and overcoming drag while progressing on the horizontal (this analysis is reviewed in the Appendix). The sum of the two efforts is minimal when the two efforts are of the same size. In this regime the body-mass scaling relations (slope and intercept) of animal locomotion are predicted and are in agreement with the measurements of swimmers, runners and fliers over the body mass (M) range 10^{-6} – 10^3 kg:

(i) travel speeds (V) proportional to $M^{1/6}$, where M is the body mass, ρ is the body density and g is gravitational acceleration, for example, for running on soft ground and swimming:

$$V \sim g^{1/2} \rho^{-1/6} M^{1/6}, \quad (1)$$

(ii) body frequencies (t^{-1}) (flapping, stride, fish tailing) proportional to $M^{-1/6}$:

$$t^{-1} \sim g^{1/2} \rho^{1/6} M^{-1/6}, \quad (2)$$

(iii) body force (F) scale equal to M :

$$F \sim Mg, \quad (3)$$

(iv) food requirement (useful energy, W) per distance traveled (L_x), which is proportional to M :

$$\frac{W}{L_x} \sim Mg. \quad (4)$$

The corresponding scaling laws for flying and running with air drag are similar to Eqns 1–4. These relations are accurate within a dimensionless factor of order 1, as they were derived based on scale analysis. In spite of this built-in approximation, they agree well with the large body of experimental data available (Bejan and Marden, 2006).

MATERIALS AND METHODS

Speed and body mass

We used this constructal framework to examine the evolution of speeds in modern athletics: the evolution of the sport (winning speeds and body metrics), not the evolution of the athletes. We focused on the two most documented probes for men, the 100m-freestyle in swimming and the 100m-dash in track. These are sprint probes, not endurance events. Sprint probes require intense expenditure of work during a relatively short period of time.

In Fig. 1A and Table 1 we see the evolution of the world speed record (V) for male 100m-freestyle swimming since 1912. Because of the theoretical scaling (i), we also researched the evolution of the body masses of the record-breaking athletes (Fig. 1B). Both V and M have been increasing in time (t). By eliminating t between Fig. 1A,B, we found Fig. 1C, which shows the evolution of V vs M .

RESULTS

There is scatter in Fig. 1C because the span of the V and M data is short, much shorter than the span of all biological cases of animal

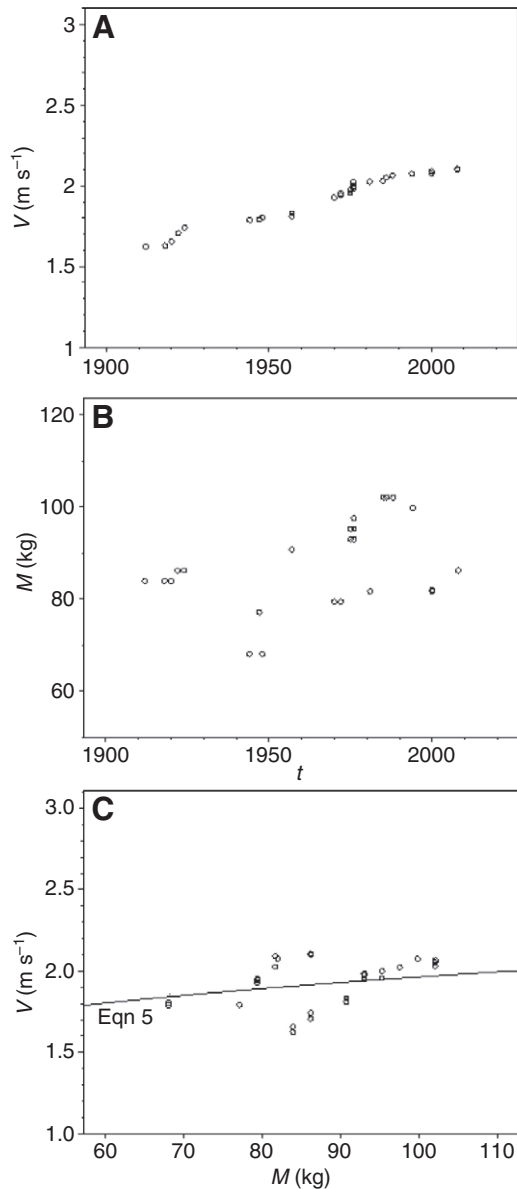


Fig. 1. Swimming world records for 100 m freestyle, men: (A) speed (V) vs time (t); (B) body mass (M) vs t ; (C) V vs M . The world record data for all the figures cover the period 1912–2008, and are listed in Table 1.

locomotion correlated in Bejan and Marden (Bejan and Marden, 2006), where the M range was 10^{-6} – 10^3 kg. The shortness of the contemporary timeframe has the effect of magnifying the scatter of the data. Several additional factors also contribute to the scatter, for example, technology (space age swimming suits, running shoes and chronometry), competition environment (state of the art aquatic and track and field venues) and changes in the rules of competition. In spite of these random variables, the evolutionary direction of animal locomotion (Eqn 1) is respected: as an average, the faster swimmers are bigger. The best fit of the (V , M) data of Fig. 1C according to the theoretical proportionality between V and $M^{1/6}$ (cf. Eqn 1) is:

$$V \sim 0.72 M^{1/6}, \quad (5a)$$

where V and M are expressed in m s⁻¹ and kg, respectively. Eqn 5a was obtained by power law regression, with $R^2=0.171$. The P -value

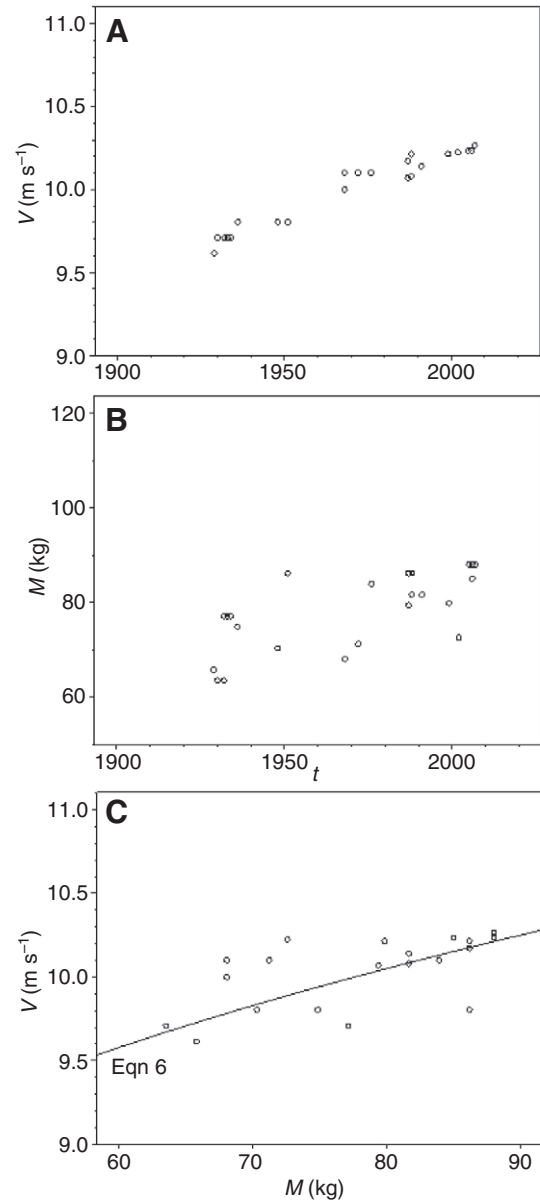


Fig. 2. Running world records for 100 m dash, men: (A) speed (V) vs time (t); (B) body mass (M) vs t ; (C) V vs M . The world record data for all the figures cover the period 1929–2008 and are listed in Table 2.

is 0.028, and because it is less than 0.05, the correlation shown in Eqn 5a is statistically significant (Soong, 2004; Vogt, 2005).

The P -value is even smaller if we correlate the V – M data of Fig. 1C with a more general power law $V=am^b$, in which the constants a and b can be optimized. The best fit of this kind is:

$$V \sim 0.68 M^{0.23}, \quad (5b)$$

for which the P -value is 0.023 and $R^2=0.19$. Note the slight difference between the exponent 0.23 and the theoretical exponent $1/60.17$. The scaling (Eqn 5b) represents a steeper increase in V with M than in the broad-range animal scaling (Eqn 1). Numerically, the two formulas (Eqn 5b and Eqn 1) agree in the M range of humans, and for this reason the V – $M^{1/6}$ scaling of Eqn 5a is sufficient for concluding that the animal scaling (Eqn 1)

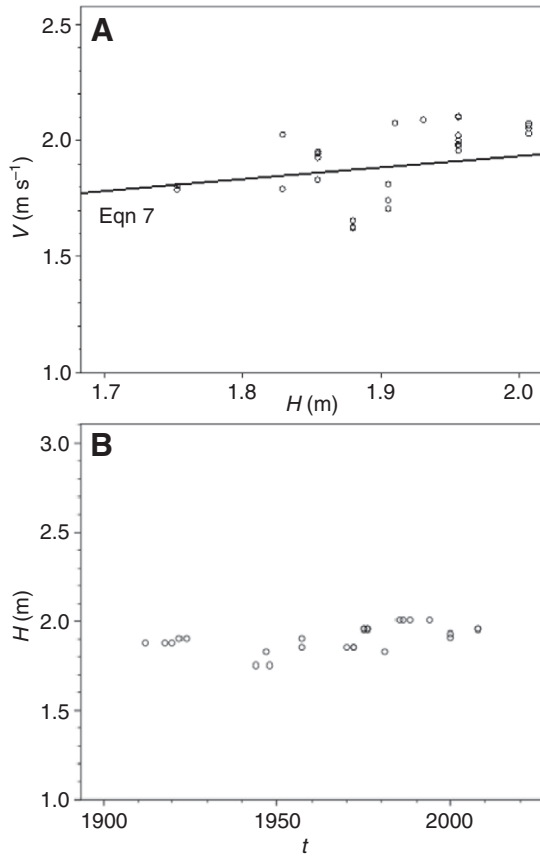


Fig. 3. Swimming world records for 100 m freestyle, men: (A) Speed (V) vs body height (H); (B) H vs time (t).

manifests itself in the evolution of swimming speeds vs mass among record holders.

For 100 m dash (Fig. 2), the trend, scatter and conclusion are the same. Eliminating t between Fig. 2A,B, we arrived at Fig. 2C. The power law regression equation for the data in Fig. 2C is:

$$V \sim 4.85 M^{1/6}, \quad (6)$$

with $R^2=0.364$. The P -value is 0.007, again indicating statistical significance.

Noteworthy are the factors 0.72 and 4.85 in Eqns 5a and 6, respectively. These factors of order 1 agree with the theoretical scaling (Eqn 1), which after substituting the values for g and ρ yields for swimming and running $V \sim 1 \times M^{1/6}$. Here '1' is the intercept of the line plotted as $\log V$ vs $\log M$. Also noteworthy is that the factor for running (4.85) is greater than the factor for swimming (0.72). This also agrees with the manner in which the empirical factor (not shown in Eqn 1 but reported in the Appendix) differentiates between the power-law correlations of animal speed data for runners and swimmers (Bejan and Marden, 2006).

Body height

The conclusion that body size has an effect on speed, Eqns 5a and 6, agrees fully with the doctrine of animal scaling (Bejan, 2000; Hoppeler and Weibel, 2005; Weibel, 2000). Size can be expressed not only as M but also as body height (H). In the scale analysis that led to Eqns 1–4, the body was modeled in the simplest possible way: with one length scale, which meant that $M \sim \rho L_b^3$, where L_b is the lone length scale. Accordingly, the theoretical speed V of Eqn 1

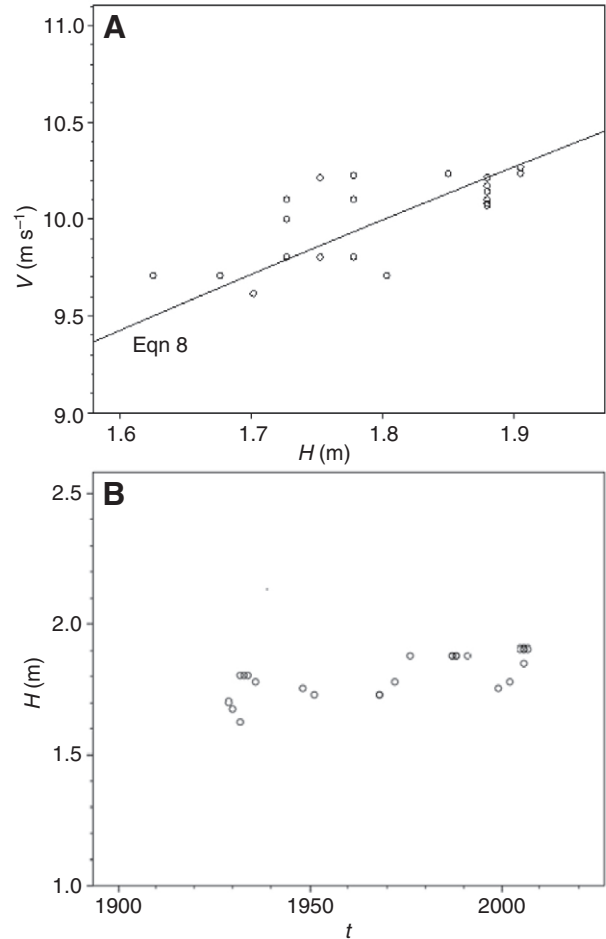


Fig. 4. Running world records for 100 m dash, men: (A) Speed (V) vs body height (H); (B) H vs time (t).

should be proportional to $L_b^{1/2}$. In Fig. 3A and Fig. 4A we plotted the data of Tables 1 and 2 by using the H of the athletes as the length scale L_b . We determined the best correlations of type $V \sim L_b^{1/2}$, based on power law regression:

$$V \sim 1.37 L_b^{1/2} \text{ (100 m freestyle)}, \quad (7)$$

$$V \sim 7.45 L_b^{1/2} \text{ (100 m dash)}, \quad (8)$$

where L_b is expressed in meters, $R^2=0.248$ and 0.433 for Eqns 7 and 8, respectively. The corresponding P -values are 0.009 and 0.001, respectively; thus, indicating statistical significance for both correlations. The H data of Tables 1 and 2 are plotted vs t in Fig. 3B and Fig. 4B, respectively.

The proportionality between speeds and body length raised to the power 1/2 (Eqns 7 and 8) suggests a simpler way to derive the speed–mass scaling rule (Eqn 1), much simpler than the analysis shown in the Appendix. During each cycle of locomotion the body falls from a height of order L_b . The time scale of the fall is of order $t \sim (L_b/g)^{1/2}$. The body falls forward to a distance of order L_b ; therefore, the horizontal velocity scale is $V \sim L_b/t \sim (gL_b)^{1/2}$. Combining this V scale with $L_b \sim (M/\rho)^{1/3}$ we arrive at Eqn 1.

Body slenderness

A body model that is more realistic than the single-scale model is a cylinder of height H and diameter (width) L . The M in this model

Table 1. Men's 100 m freestyle world records

Year	Name	Time (s)	Height (m)	Mass (kg)	Slenderness
1912	Duke Kahanamoku	61.6	1.88	83.9	7.88
1918	Duke Kahanamoku	61.4	1.88	83.9	7.88
1920	Duke Kahanamoku	60.4	1.88	83.9	7.88
1922	Johnny Weissmuller	58.6	1.91	86.2	7.94
1924	Johnny Weissmuller	57.4	1.91	86.2	7.94
1944	Alan Ford	55.9	1.75	68.0	7.88
1947	Alex Jany	55.8	1.88	77.1	7.89
1948	Alan Ford	55.4	1.75	68.0	7.88
1956	John Henricks	55.4	1.80	81.6	7.74
1957	John Devitt	55.2	1.91	90.7	7.43
1957	John Devitt	54.6	1.91	90.7	7.43
1961	Steve Clarke	54.4	1.83	75.9	7.96
1968	Michael Wenden	52.2	1.85	78.2	7.97
1970	Mark Spitz	51.9	1.85	79.4	7.95
1972	Mark Spitz	51.47	1.85	79.4	7.95
1972	Mark Spitz	51.22	1.85	79.4	7.95
1975	James Montgomery	51.12	1.96	93.0	7.95
1975	Andrew Cohen	51.11	1.96	95.3	7.85
1975	James Montgomery	50.59	1.96	93.0	7.94
1976	James Montgomery	50.39	1.96	93.0	7.94
1976	James Montgomery	49.99	1.96	93.0	7.94
1976	Jonty Skinner	49.44	1.96	97.5	7.76
1981	Rowdy Gains	49.36	1.83	81.6	7.67
1985	Matt Biondi	49.24	2.00	102.1	7.88
1986	Matt Biondi	48.74	2.00	102.1	7.88
1988	Matt Biondi	48.42	2.00	102.1	7.88
1994	Alexander Popov	48.21	2.00	99.8	7.97
2000	Michael Klim	48.18	1.91	82.0	8.16
2000	Pieter van den Hoogenband	47.84	1.93	81.6	8.31
2008	Alain Bernard	47.6	1.96	86.2	8.26
2008	Alain Bernard	47.5	1.96	86.2	8.26
2008	Eamon Sullivan	47.24	1.90	78.2	8.29
2008	Alain Bernard	47.2	1.96	86.2	8.26
2008	Eamon Sullivan	47.05	1.90	78.2	8.29

Table 2. Men's 100 m dash world records

Year	Name	Time (s)	Height (m)	Mass (kg)	Slenderness
1929	Eddie Tolan	10.4	1.70	65.8	7.61
1930	Percy Williams	10.3	1.68	63.5	7.63
1932	Eddie Tolan	10.3	1.70	65.8	7.67
1932	Ralph Metcalfe	10.3	1.80	77.1	7.73
1933	Ralph Metcalfe	10.3	1.80	77.1	7.73
1934	Ralph Metcalfe	10.3	1.80	77.1	7.73
1936	Jesse Owens	10.2	1.78	74.8	7.68
1948	Barney Ewell	10.2	1.75	70.3	7.75
1951	Emmanuel McDonald Bailey	10.2	1.73	86.2	6.85
1968	Charles Greene	10	1.73	68.0	7.71
1968	Charles Greene	9.9	1.73	68.0	7.71
1972	Eddie Hart	9.9	1.78	71.2	7.87
1976	Harvey Glance	9.9	1.88	83.9	7.88
1987	Carl Lewis	9.93	1.88	81.6	7.99
1987	Ben Johnson	9.83	1.88	86.2	7.78
1988	Ben Johnson	9.79	1.88	86.2	7.78
1988	Carl Lewis	9.92	1.88	81.6	7.99
1991	Carl Lewis	9.86	1.88	81.6	7.99
1999	Maurice Greene	9.79	1.75	79.8	7.28
2002	Tim Montgomery	9.78	1.78	72.6	7.80
2005	Asafa Powell	9.77	1.91	88.0	7.85
2006	Justin Gatlin	9.77	1.85	85.0	7.65
2006	Asafa Powell	9.77	1.91	88.0	7.85
2006	Asafa Powell	9.77	1.91	88.0	7.85
2007	Asafa Powell	9.74	1.91	88.0	7.85
2008	Usain Bolt	9.69	1.96	86.0	8.29

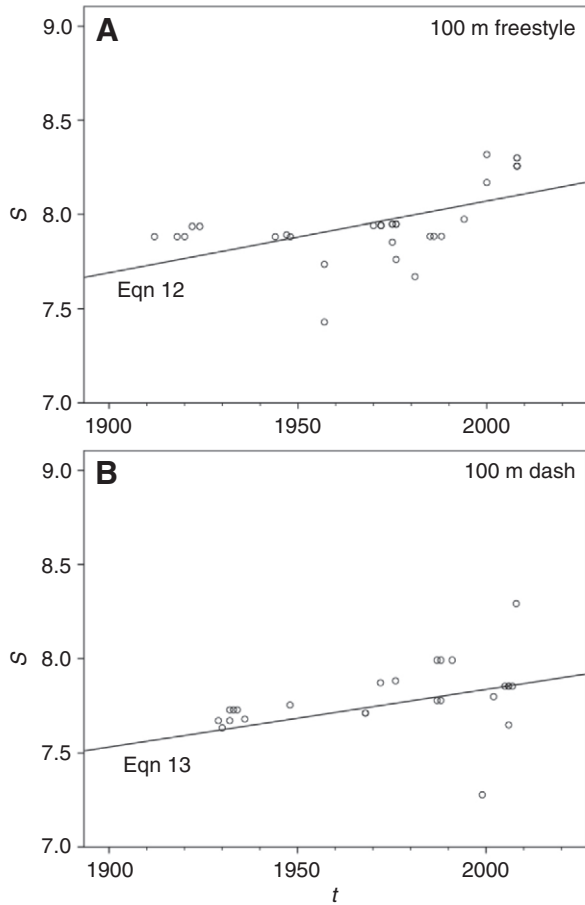


Fig. 5. The evolution of the slenderness of record holders over time: (A) 100 m freestyle, men; (B) 100 m dash, men.

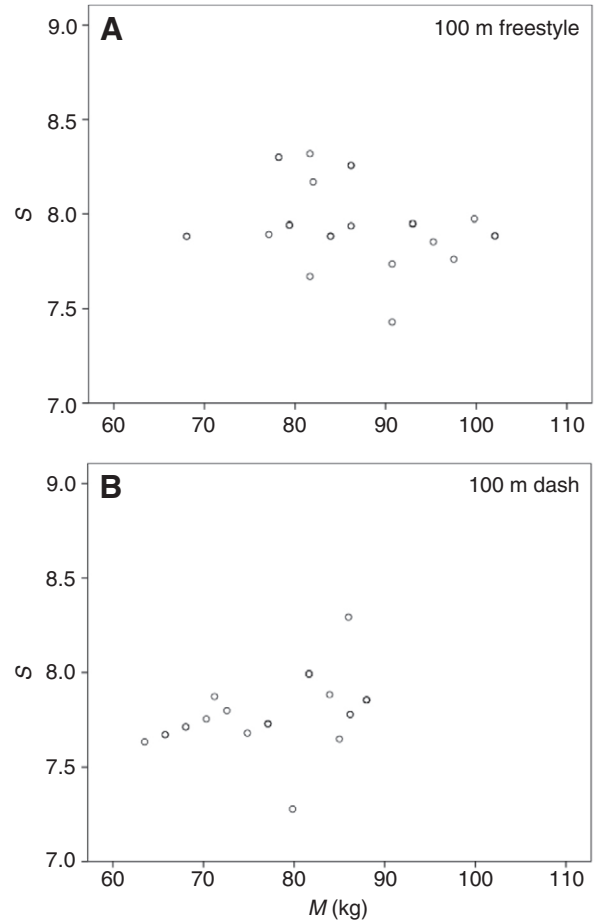


Fig. 6. The slenderness of record holders vs body mass: (A) 100 m freestyle, men; (B) 100 m dash, men.

is $M=\rho(\pi/4)L^2H$. We used the body density ($\rho\sim 1000\text{kgm}^{-3}$), the recorded M and the recorded H to calculate the athlete's width scale $L=(4M/\rho\pi H)^{1/2}$. We then used the recorded H and the calculated L to define the slenderness (S) of the body:

$$S = \frac{H}{L} = \left(\frac{\pi\rho H^3}{4M} \right)^{1/2} \quad (9)$$

The S values calculated in this manner are reported for each athlete in Tables 1 and 2. Their significance becomes apparent if we combine this two-scale body model with the locomotion model proposed at the end of the preceding section.

In swimming, the vertical length scale is L , the time scale of the fall is $(L/g)^{1/2}$ and the forward speed is of order $V\sim(gL)^{1/2}$. Omitting factors of order 1 (such as $\pi/4$), we combine the mass scale ($M\sim\rho L^2H$) with $S=H/L$ and obtain $L\sim(M/\rho)^{1/3}S^{-1/3}$ and:

$$V_{\text{swim}} \sim g^{1/2} \rho^{-1/6} M^{1/6} S^{-1/6} \quad (10)$$

In running, the vertical length scale is H , and the corresponding scales are $t\sim(H/g)^{1/2}$, $V\sim(gH)^{1/2}$ and $H\sim(M/\rho)^{1/3}S^{2/3}$. The speed-mass relation that replaces Eqn 1 is:

$$V_{\text{run}} \sim g^{1/2} \rho^{-1/6} M^{1/6} S^{1/3} \quad (11)$$

The S effect differentiates between running and swimming. Dividing Eqns 11 and 10 we anticipate $V_{\text{run}}/V_{\text{swim}}\sim S^{1/2}$, which is a

number of the same order as the ratio between Eqns 6 and 5a. The two-scale model also suggests that from among athletes with the same mass, the ones with larger S values are more likely to run fast. In swimming, the S effect is the opposite but weaker: swimmers would be slightly faster if more robust (smaller S).

Fig. 5A,B show the S data plotted vs t , and indicate a weak progress toward larger S values for both swimming and running. The best linear fits for the two sets of data are:

$$S = 0.48 + 0.0038t \text{ (100 m freestyle)}, \quad (12)$$

$$S = 1.7 + 0.0031t \text{ (100 m dash)}, \quad (13)$$

where t is the year, $R^2=0.31$ and 0.13 , and $P=0.001$ and 0.07 , respectively.

The same S data are plotted against M in Fig. 6A,B. The data are too sparse to yield statistically significant correlations; however, qualitatively they suggest a slight increase in S vs M for running and a slight decrease in S vs M for swimming.

DISCUSSION

The scaling trends revealed by the speed data suggest that speed records will continue to be dominated by heavier and taller athletes. This trend is due to the scaling rules of animal locomotion, not to the contemporary increase in the average body size of humans. The

mean height of humans has increased by roughly 5 cm from 1900 to 2002 (Plastic Soldier Review, 2002). During the same century, the mean height of champion swimmers and runners has increased by 11.4 cm and 16.2 cm, respectively (Fig. 3C, Fig. 4C).

The insight gained in this paper allows us to speculate what the running speeds might have been in ancient Greece and the Roman Empire. There is no record of what the winning speeds were then, because the competition was for winning the race, not for breaking a time record. Chronometry did not exist. In antiquity body masses were roughly 70% of what they are today (Plastic Soldier Review, 2002; Hpathy 2009; National Health and Nutrition Examination Survey, 1999). According to Eqn 6, this means that speeds were lower by a factor of roughly $(0.7)^{1/6}=0.94$. In other words, if the 100m dash in military training today is won in 13 s, 2000 years ago it would have been won in ~14 s.

This insight also teaches us why certain training techniques are successful in high-performance sports. For example, in modern speed swimming, the doctrine holds that the swimmer must raise his body to the highest level possible above the water. Two explanations are given for this swimming doctrine: air drag is much smaller than water friction, and the water wave generated by the body propels the body better (Collela, 2009). The doctrine is correct but for a different reason, which is evident in Eqn 7. When the body is high above the water it falls faster (and forward) when it reaches the water line. For the same reason, the speeds of all water waves exhibit the same scale as in Eqn 7, in which L_b is the length scale of the wave (Prandtl, 1969). The crest of the wave falls with a speed of order $(g L_b)^{1/2}$, which becomes visible as the forward speed of the traveling wave.

CONCLUSION

In the future, the fastest athletes can be expected to be heavier and taller. If the winners' podium is to include athletes of all sizes, then speed competitions might have to be divided into weight categories. This is not at all unrealistic in view of the body force scaling (Eqn 3), which was recognized from the beginning in the structuring of modern athletics. Larger athletes lift, push and punch harder than smaller athletes, and this led to the establishment of weight classes for weight lifting, wrestling and boxing. Larger athletes also run and swim faster.

APPENDIX

Here is a brief summary of the scale analysis of animal locomotion, which leads to Eqns 1–4. It was first done for flying (Bejan, 2000) and then generalized to all locomotion: running, flying and swimming (Bejan and Marden, 2006).

The animal body has a single length scale (L_b). Its mass scale is $M \rho L_b^3$. Locomotion is a rhythm – a sequence of cycles. During each cycle the body must perform work in two ways, in the vertical direction (W_1) and in the horizontal direction (W_2). Both W_1 and W_2 are destroyed. The vertical work is necessary in order to lift the body to a height of order L_b :

$$W_1 \sim Mg L_b. \quad (\text{A1})$$

The horizontal work is necessary in order to push through and penetrate the surrounding medium. If the body length and speed scales (i.e. the Reynolds number) are large enough, then the horizontal work is of order:

$$W_2 \sim F_{\text{drag}} L_x, \quad (\text{A2})$$

$$F_{\text{drag}} \sim \rho_m V^2 L_b^2 C_D, \quad (\text{A3})$$

where F_{drag} is the drag force, ρ_m is the density of the medium, $C_D \sim 1$ is the drag coefficient and L_x is the distance traveled during the

cycle. Together, these scales allow us to estimate the total work per distance travelled:

$$\frac{W_1 + W_2}{L_x} \sim \frac{Mg L_b}{L_x} + \rho_m V^2 L_b^2. \quad (\text{A4})$$

The time scale of the cycle is the Galilean time of free fall from the height L_b :

$$t \sim \left(\frac{L_b}{g} \right)^{1/2}. \quad (\text{A5})$$

The horizontal travel during the cycle is $L_x \sim Vt$, and Eqn A4 becomes:

$$\frac{W_1 + W_2}{L_x} \sim Mg^{3/2} \frac{L_b^{1/2}}{V} + \rho_m L_b^2 V^2. \quad (\text{A6})$$

The right side is a sum of two terms combined as $A/V+B V^2$, where A and B are two constants and V may vary. This sum is minimal when $V \sim (A/B)^{1/3}$, which yields:

$$V \sim \left(\frac{\rho}{\rho_m} \right)^{1/3} g^{1/2} \rho^{-1/6} M^{1/6}. \quad (\text{A7})$$

The frequency associated with this cycle is $\tau^{-1} \sim (g/L_b)^{1/2}$ or:

$$\tau^{-1} \sim g^{1/2} \rho^{1/6} M^{-1/6}. \quad (\text{A8})$$

The necessary body force scale F is dictated by the lifting work $W_1 \sim F L_b \sim Mg L_b$ therefore:

$$F \sim Mg. \quad (\text{A9})$$

The minimum work per distance traveled is obtained by substituting Eqn A7 and $L_b \sim (M/\rho)^{1/3}$ into Eqn A6:

$$\left(\frac{W_1 + W_2}{L_x} \right)_{\min} \sim \left(\frac{\rho_m}{\rho} \right)^{1/3} Mg. \quad (\text{A10})$$

In conclusion, the scaling relations (Eqns 1–4) have been derived here in Eqns A7–A10. The modifying factor $(\rho_m/\rho)^{1/3}$ depends on the medium. In flying, the ρ_m (air) is roughly equal to $\rho/10^3$, and the factor $(\rho_m/\rho)^{1/3}$ is close to 1/10. In swimming, the ρ_m (water) is the same as the body density, and the factor $(\rho_m/\rho)^{1/3}$ is 1. In running, the modifying factor is between 1/10 and 1, and depends on the nature of the running surface and the speed. For example, running through snow, mud and sand is represented by a $(\rho_m/\rho)^{1/3}$ value close to 1. Running at high speed on a dry surface is represented more closely by a $(\rho_m/\rho)^{1/3}$ factor similar to the one that represents flying.

In summary, the effect of the factor $(\rho_m/\rho)^{1/3}$ is weak and of order 1, and for this reason it was left out of Eqns 1 and 4. Important to note is that $(\rho_m/\rho)^{1/3}$ differentiates between locomotion media in an unmistakable direction: if M is fixed, speeds increase in the direction sea \rightarrow land \rightarrow air (cf. Eqn A7); the work requirement decreases in the same direction (cf. Eqn A10). The animal speeds collected over the $M=10^{-3}$ – 10^3 kg range in fig. 2 of Bejan and Marden (Bejan and Marden, 2006) confirm the differentiating effect of the surrounding medium. Each cloud of data is approximated by:

$$V \sim 10M^{1/6} \text{ (flyers)}, \quad (\text{A11})$$

$$V \sim 4M^{1/6} \text{ (runners)}, \quad (\text{A12})$$

$$V \sim 1M^{1/6} \text{ (swimmers)}. \quad (\text{A13})$$

It can be verified that Eqn A11–A13 agree with Eqn 1 within a factor of order 1. They also agree with Eqns 5 and 6.

The data collected in fig. 2 of Bejan and Marden (Bejan and Marden, 2006) also confirm that the medium factor $(\rho_m/\rho)^{1/3}$ does not have an effect on the frequencies (f^{-1}) and forces (F) of flyers, runners and swimmers. This is in accordance with Eqns A8 and A9, in which $(\rho_m/\rho)^{1/3}$ is not present.

LIST OF ABBREVIATIONS

C_D	drag coefficient
F	force, N
F_{drag}	drag force, N
g	gravitational acceleration, m s^{-2}
H	body height, m
L	body width, m
L_b	single body length scale, m
L_x	distance traveled, m
M	body mass, kg
P	probability of true null hypothesis
R^2	coefficient of determination
S	body slenderness, H/L
t	period, s
t	time, years
f^{-1}	frequency, s^{-1}
V	speed, m s^{-1}
W	work, useful energy, J
W_1	work done vertically, J
W_2	work done horizontally, J
ρ	body density, kg m^{-3}
ρ_m	density of the medium, kg m^{-3}

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