

Limits to running speed in dogs, horses and humans

Mark W. Denny

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There was an error published in *J. Exp. Biol.* **211**, 3836-3849.

In Eqn 3 (p. 3838), the denominator in the ratio contained within the brackets was written incorrectly. The correct version of the equation is shown below.

$$V(y) = mn + (mx - mn) \left(\frac{\exp[k(y - t)]}{1 + \exp[k(y - t)]} \right), \quad (3)$$

The author apologises to readers for this error but confirms that the correct equation was used in the analyses and therefore the error does not affect the results or conclusions of the paper.

Limits to running speed in dogs, horses and humans

Mark W. Denny

Hopkins Marine Station of Stanford University, Pacific Grove, CA 93950, USA

e-mail: mwdenny@stanford.edu

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SUMMARY

Are there absolute limits to the speed at which animals can run? If so, how close are present-day individuals to these limits? I approach these questions by using three statistical models and data from competitive races to estimate maximum running speeds for greyhounds, thoroughbred horses and elite human athletes. In each case, an absolute speed limit is definable, and the current record approaches that predicted maximum. While all such extrapolations must be used cautiously, these data suggest that there are limits to the ability of either natural or artificial selection to produce ever faster dogs, horses and humans. Quantification of the limits to running speed may aid in formulating and testing models of locomotion.

Key words: running, terrestrial locomotion, horse, dog, thoroughbred, greyhound, track and field, speed limits, maximum speed, evolution, world record, human.

INTRODUCTION

Legged locomotion is a complicated process (for reviews, see Alexander, 2003; Biewener, 2003). As it walks or runs, an animal periodically accelerates both its limbs and its center of gravity. These accelerations require the coordinated application of forces by muscles and skeletal 'springs', and the mechanical and neural coordination of these forces can be complex. In turn, the acceleration of the body's various masses and the contraction of muscles place stresses on an organism's skeleton that can be potentially harmful. The metabolic demands of locomotion vary with the morphology, size, speed and gait of the animal.

The extent of our understanding of the complex process of legged locomotion can be assessed through a variety of metrics. One common index is a comparison between measured and predicted maximum speeds: if we understand the physiology and mechanics of locomotion in a particular animal, we should be able to accurately predict how fast that animal can run. Several approaches have been applied to this task. Maximum running speeds have been predicted based on (1) the mass of the body or of its locomotory musculature (e.g. Hutchison and Garcia, 2002; Weyand and Davis, 2005), (2) the rate at which energy can be provided to the limbs (Keller, 1973), (3) the ground force muscles can produce (e.g. Weyand et al., 2000), (4) the stiffness of the 'spring' formed by the muscles, ligaments and skeleton (e.g. Farley, 1997), (5) the aerobic capacity of the lungs and circulatory system (e.g. Jones and Lindstedt, 1993; Weyand et al., 1994), and (6) the strength of bones, ligaments and tendons (e.g. Biewener, 1989; Biewener, 1990; Blanco et al., 2003; Iriarte-Diaz, 2002). All of these factors vary with body size, limb morphology and the distance over which speed is measured.

To assess the predictive accuracy of these models, we need empirical standards with which their predictions can be compared. The more precise the models become, the more precise these standards need to be. Therein lies a problem. For extinct species [e.g. *Tyrannosaurus* (Hutchison and Garcia, 2002)] it is impossible to measure speed directly. For many extant species, maximum running speed has been measured for only a few individuals and often under less than ideal conditions. Even under the best of

circumstances the accuracy of speed measurements is often highly questionable (e.g. Alexander, 2003).

Human beings provide an illustrative case study. In many respects, humans are ideal experimental animals for the measurement of maximum speeds. They are intelligent and highly motivated to accomplish a given task, and a vast number of speed trials (competitive races) have been conducted over the years at a wide variety of distances. However, despite this wealth of experimental data, it has proven difficult to quantify the maximum running speeds of humans. In large part, this difficulty is due to the fact that measured speeds have changed through time. For example, the world record speed for men running 1500 m is 14% faster today than it was a century ago (Quercetani and Magnusson, 1988; Lawson, 1997) (International Association of Athletics Federations, <http://www.iaaf.org/statistics/records/inout=O/index.html>), and speed in the marathon (42.2 km) is nearly 23% faster than it was in 1920. Increases in speed among women are even more dramatic: 21% faster in the 1500 m race since 1944 and more than 60% faster in the marathon since 1963. If maximum running speed changes through time, it is difficult to use it as a standard for comparison.

The same problem applies to other well-studied species. Horses and dogs have raced competitively for centuries, and one might suppose that their maximum speeds would be well established. But, as with humans, the speed of horses and dogs has increased through time. The winning speed in the Kentucky Derby (a race for thoroughbred horses) increased by more than 13% in the 65 years from 1908 to 1973. The winning speed in the greyhound English Grand National has increased by nearly 15% in the 80 years since its inception in 1927.

The temporal variability in maximum running speeds of dogs, horses and humans raises two central questions. (1) Is there a definable maximum speed for a species running a given distance? The upward trend through time in race speeds for dogs, horses and humans demonstrates that advances in training and equipment, and evolution of the species itself (through either natural selection or selective breeding), can increase running performance. But improvements of the magnitude observed over the last century cannot continue indefinitely: for any given distance, any species

will eventually reach its limits. However, it remains to be seen whether this limit can be reliably and accurately measured. (2) If there is a limit to speed, how does it compare with the speed of extant animals? Greyhounds and thoroughbreds have been the subject of intensive selective breeding. How successful has this breeding been in producing fleet animals? Humans are not bred for speed (at least not in the formal fashion of dogs and horses), but what they lack in breeding they have attempted to make up for with improvements in training, nutrition and equipment, and through the use of performance-enhancing drugs. How successful have these efforts been in producing faster humans?

Past attempts at predicting progress in running performance (primarily in humans) have been less than satisfying. Early attempts documented historical trends and extrapolated these trends linearly into the future (e.g. Whipp and Ward, 1992; Tatem et al., 2004). However, these analyses provide no hint of the absolute limits that must exist. Taken to their logical extremes they make absurd predictions: negative race times, speeds in excess of Mach 1. More recent efforts fit historical trends to exponential and logistic equations, models capable of explicitly estimating maximum speeds (e.g. Nevill and Whyte, 2005). However, these models have been applied only to humans running a few distances and, even then, only to world records, a small subset of the available data.

Here I use the statistics of extremes and three statistical modeling approaches to estimate maximum running speeds for greyhounds, thoroughbred horses and elite human athletes.

MATERIALS AND METHODS

I define 'speed' as the average speed that an organism maintains over a fixed distance on level ground (=event distance/total elapsed time). This definition avoids the complications inherent in measuring instantaneous speed (e.g. Tibshirani, 1997). For thoroughbreds and greyhounds, extensive reliable data are available for only narrow ranges of distance (1911–2414 m for horses and 460–500 m for greyhounds), but for humans, data are available for distances varying by more than two orders of magnitude (100–42,195 m).

Historical records

Horses

Data were obtained for the US Triple Crown races (the Kentucky Derby, Preakness Stakes and Belmont Stakes), contested by 2 year olds. The Kentucky Derby has been run annually since 1875, but the current race distance (1.25 miles, 2012 m) was not set until 1896. I obtained winning race times for the years 1896–2008 from the race's official web site (www.kentuckyderby.com/2008/history/statistics). The Preakness Stakes has been run annually since 1873, and at the current race distance (1.1875 miles, 1911 m) since 1925. Data for 1925–2008 were obtained from the race's official web site (www.preakness-stakes.info/winners.php). The Belmont Stakes has been contested annually since 1867. The current race distance (1.5 miles, 2414 m) was set in 1926, and I obtained data for 1926–2008 from the official web site of the New York Racing Association (www.nyra.com/Belmont/Stakes/Belmont.shtml). Unlike racing dogs and humans, racing horses carry a jockey. The weight of the jockey and saddle in the Triple Crown races is typically 55–58 kg.

Dogs

Professional greyhound racing was established in Great Britain in 1927, and three races have been contested annually since that date (with a gap during World War II). Winning race times for three premier dog races (the English Derby, English Grand National and English Oaks, named after horse races) were obtained from

www.greyhound-data.com. The length of each race has varied occasionally; only data for races of 460–480 m were used.

Humans

The ATFS (Association of Track and Field Statisticians, 1951–2006), Quercetani and Magnusson (Quercetani and Magnusson, 1988), Magnusson and colleagues (Magnusson et al., 1991), Kök and colleagues (Kök et al., 1995; Kök et al., 1999) and the International Association of Athletics Federations (<http://www.iaaf.org/statistics/records/inout=O/index.html>; accessed June 2008) document annual world's best race times for both men and women for races varying from 100 m to the marathon (42,195 m). Unlike thoroughbreds (for which races are typically among individuals of a single year class), humans often race competitively for several years. On occasion, a single individual has recorded the world's best time in more than one year. This year-to-year connection is a problem for the methods applied here to the analysis of maximum speeds (extreme-value analysis, see below), which assume that individual data points are independent. To minimize this factor, only an individual's best time was used; other years in which that individual recorded the world's best time were deleted from the record. In cases where an individual recorded identical best times in separate years, the first occurrence of the time was used.

Data for men's races are available for some distances beginning in 1900, and for all distances from 1921. Except in the 100 m and 200 m races and the marathon (which include data from 2008), all records extend to 2007. Data for women's races from 100 m to 800 m are available from 1921 to 2007, but data for longer distances are more constrained. Data for the 1500 m race begin in 1944, with a gap from 1949 to 1966, sufficient for the present purposes. Data for the 3000 m, 5000 m and 10,000 m races are too scarce to be useful for this analysis. Records for the women's marathon are available from 1963 to 2007, and are sufficient for analysis.

Several minor adjustments and corrections were made to the human historical record (see Appendix 1).

Analytical approaches

Extreme-value analysis

Each data point in these historical race records is the annual maximum speed recorded from a select group of highly trained athletes in a given race. As such, each record is a sample of the extreme abilities of the species in question. The statistics of extremes (Gaines and Denny, 1993; Denny and Gaines, 2000; Coles, 2001; Katz et al., 2005) asserts that the distribution of these extreme values should conform asymptotically to a generalized extreme-value (GEV) distribution:

$$P(V) = \exp \left\{ - \left[1 + a \left(\frac{V-b}{c} \right) \right]^{1/a} \right\}. \quad (1)$$

Here, $P(V)$ is the probability that an annual maximum speed chosen at random is $\leq V$. The shape of this cumulative probability curve is set by three parameters: a , a shape parameter; b , a location parameter; and c , a scale parameter. Parameters a and b can take on any value; c is constrained to be >0 . If $a \geq 0$, the shape of the distribution of extreme values is such that there is no defined limit to the extremes that can potentially be reached (Coles, 2001). In contrast, if $a < 0$, $P=1$ when $V=b-(c/a)$. In such a case, the distribution of extreme values has a defined absolute maximal value:

$$V_{\max} = b - \frac{c}{a}. \quad (2)$$

Because of this ability to define and quantify absolute maxima, the statistics of extremes is a promising approach for the study of maximum running speeds. All extreme-value analyses were carried out using extRemes software (E. Gilleland and R. W. Katz, NCAR Research Applications Laboratory, Boulder, CO, USA), an implementation in the R language of the routines devised by Coles (Coles, 2001). When fitting Eqn 1 to data, there is inevitably some uncertainty in the estimated value of each parameter and of V_{\max} . Confidence limits for these values were determined using the profile likelihood method (Coles, 2001).

Method 1: no trend

Appropriate application of Eqn 1 to the estimate of maximum speeds depends on whether or not there is a trend in race data, either with time or with population size. For some races, speeds appear to have plateaued in recent years. The existence and extent of such a plateau was determined by sequentially calculating the correlation between year and maximum speed starting with data for the most recent 30 years and then extending point by point back in time. If there was no statistically significant trend in speed, a plateau was deemed to exist, and the beginning of the plateau was taken as the year associated with the lowest correlation coefficient. For the data in such a plateau, application of Eqn 1 is straightforward: parameters a , b and c are chosen to provide the best fit to the raw speed data in the plateau, the degree of fit being judged by a maximum likelihood criterion. If a is significantly less than 0 (that is, if its upper 95% confidence limit is <0), the absolute maximum is calculated according to Eqn 2 and confidence limits for this estimate are obtained. In cases where $a \geq 0$, no absolute maximum value can be determined. In these cases, I arbitrarily use the estimated maximum value for a return time of 100 years as a practical substitute for the absolute maximum.

Method 2: the logistic model

For cases in which trends are present in race data, the trend must first be modeled before the analysis of extremes can begin. Two models were used. The first model is the logistic equation mentioned in the Introduction:

$$V(y) = mn + (mx - mn) \left(\frac{\exp[k(y - t)]}{\exp[1 + k(y - t)]} \right), \quad (3)$$

which provides a flexible means to model values that, through time, approach an upper limit (Nevill and Whyte, 2005). Eqn 3 is a natural fit to a basic assumption made here: that an absolute limit must exist to the speed at which animals can run. In Eqn 3, $V(y)$ is the fastest speed recorded in year y , mn is the model's minimum fastest speed and mx is the model's maximum fastest speed (the parameter of most interest in the current context). k is a shape parameter determining how rapidly values transition from minimum to maximum, and t is a location parameter that sets the year at which the rate of increase is most rapid. Note that although the logistic equation incorporates the assumption that a maximum speed exists, it does not assume that speeds measured to date are anywhere near that maximum.

Historical race data were fitted to Eqn 3 using a non-linear fitting routine with a least-squares criterion of fit (Systat, SPSS, Chicago, IL, USA), providing an estimate ($\pm 95\%$ confidence limit) for mx . Note that the confidence limits for mx indicate the range in which we expect to find this parameter of the model, but that this range does not necessarily encompass the variation of the data around the model. A heuristic example is shown in Fig. 1. For this example, I created a hypothetical set of race data

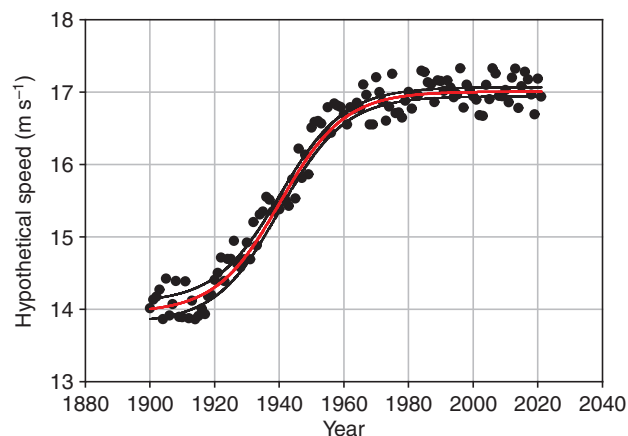


Fig. 1. Hypothetical data and the fit to them using the logistic equation (Eqn 3). The red line is the best-fit logistic model, and the black lines are the confidence limits on that fit, drawn using the best-fit values for the shape parameter k and the location parameter t and the 95% ranges for the minimum fastest speed mn and the maximum fastest speed mx . Note that the model's confidence interval does not incorporate the highest speeds.

by specifying a logistic curve ($mn=14$, $mx=17$, $k=0.1$, $t=1940$) for $y=1900$ to 2020 and adding to each deterministically modeled annual maximum speed a random speed selected uniformly from the range -0.35 to 0.35 m s^{-1} . When provided with these hypothetical data, the fitting routine very accurately estimated the true values for the parameters of the model, but the estimated mx (17.00 m s^{-1} , with 95% confidence limits $\pm 0.06 \text{ m s}^{-1}$) does not include the maximum speed in the data set (17.32 m s^{-1}). To accurately estimate the overall maximum (rather than the maximum of the trend), it is necessary to characterize the effect of variation around the fitted logistic model.

To do so, the temporal trend in extreme values is first incorporated into the analysis by subtracting the best-fit logistic model from the measured annual maximum race speeds (Gaines and Denny, 1993; Denny and Gaines, 2000). The distribution of the resulting deviations – trend-adjusted extreme values – is then analyzed using Eqn 1. If a for the distribution of trend-adjusted extremes is <0 , there is a defined absolute maximum deviation from the logistic model (Eqn 2), and this absolute maximum deviation is added to the estimated upper limit of the logistic model (the best-fit mx) to yield an estimate of the overall absolute maximum speed. As before, if $a \geq 0$, I estimate a practical maximum by adding the 100 year return value to mx . In both cases, 95% confidence limits on the predicted maximum deviation give an indication of the statistical confidence in the overall estimate of maximum speed.

Unlike the logistic analysis of Nevill and Whyte (Nevill and Whyte, 2005), which used only world-record speeds, the analysis here uses the much more extensive measurements of annual maximum speed. The variation of these annual maxima around the underlying trend provides insight into the random variation in maximum speed present in the population of animals under consideration. And unlike the analysis of data in just the plateau portion of a record (method 1), analysis using deviations from the logistic equation utilizes all the data in the historical record.

Method 3: population-driven analysis

The larger the population from which racing dogs, horses or humans are selected each year, the higher the probability that an

exceptional runner will be found by chance alone (Yang, 1975). Thus, if population size grows through time, maximum recorded speed may increase. To assess the effect of this interaction between speed and population, I used the following analysis, illustrated here using human females as an example.

I begin by assuming that there exists an idealized distribution of the speeds at which individual women run a certain distance. Every time a woman runs a race at that distance, her speed is a sample from this distribution. I then suppose that in a given year S_0 women run the race, and we record V , the fastest of these speeds. V is thus one sample from a different distribution – the distribution of maximum speeds for women running this distance. A different random sample of S_0 individual speeds would probably yield a different V . In this fashion, repeated sampling provides information about the distribution of maximum speeds. Our job is to analyze the measured values of V to ascertain whether the distribution of these maximum speeds has a defined upper bound. Statistical theory of extremes (Coles, 2001) suggests that if we confine our exploration to maximum speeds above a sufficiently high threshold u , the cumulative distribution of our sampled maxima asymptotes to the generalized Pareto family of equations (GPE):

$$G(V) = \left[1 + \varepsilon \left(\frac{V - u}{\sigma} \right) \right]^{-1/\varepsilon} \quad (4)$$

Here $G(V)$ is the probability that the maximum speed of a sample of S_0 women is $>V$.

Parameters ε and u can take on any value, σ is constrained to be >0 . Note, if $\varepsilon < 0$, $G(V) = 0$ when $V = u - (\sigma/\varepsilon)$. In other words, if ε is negative:

$$V_{\max} = u - \frac{\sigma}{\varepsilon} \quad (5)$$

Thus, if we have empirical data for V and $G(V)$, we can estimate u , σ and ε , and potentially calculate V_{\max} (Fig. 2A).

We have historical data for annual maximum speeds, but how can we estimate the corresponding probabilities, G ? Here population size comes into play. The larger the population of running women available to be sampled in a given year, the greater the chance of having a woman run at an exceptionally high speed. Put another way, the larger the S , the population of female runners, the faster the V that we are likely to record. The faster the V we record in a large sample, the lower the probability that we would have exceeded that high V in a small sample. [Recall that $G(V)$ is defined specifically for sample size S_0 .] Thus, V recorded at large S should have a relatively low G . The sense of this relationship between population size and $G(V)$ is depicted in Fig. 2A (note the ordinate on the right). The theoretical relationship between population size and annual maximum velocity is described by a modification of the GPE:

$$\frac{N_0}{N(V)} = \left[1 + \varepsilon \left(\frac{V - u}{\sigma} \right) \right]^{1/\varepsilon} \quad (6)$$

Here $N(V)$ is world population size in the year in which V was measured and N_0 is world population size at the beginning of the historical record. For a derivation of this equation, see Appendix 2.

Note that total world population is used in Eqn 6 solely as an index of the population of runners. The actual number of runners is some unknown fraction f of the total population. Both as a practical matter and for the sake of simplicity, I assume that f is constant through time, in which case it cancels out of the equation when the ratio of population sizes is taken. I recognize the likelihood that, in

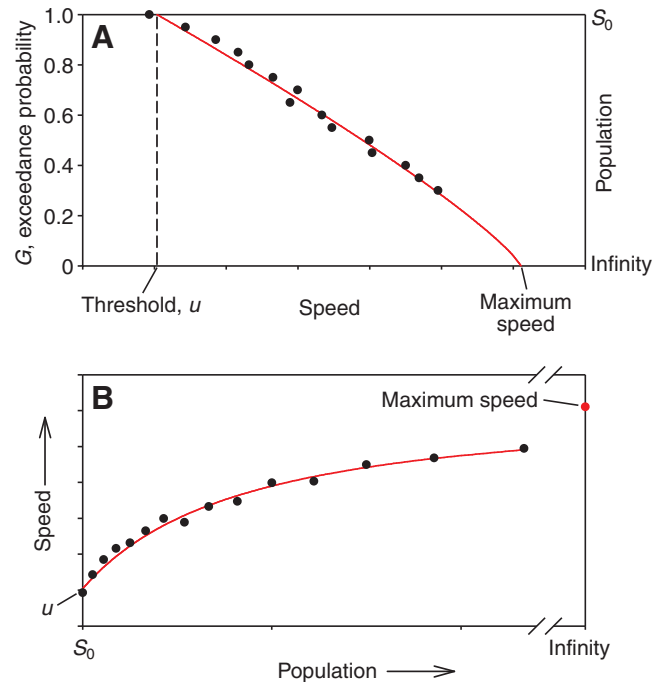


Fig. 2. (A) The generalized Pareto equation (GPE, Eqn 6) can be used to estimate absolute maximum speed. Hypothetical measured data are shown as black dots, and the best-fit GPE fitted to these data (the red line) can be extrapolated to an exceedance probability of $G=0$, thereby estimating the maximum possible speed. Note from the ordinate on the right that $G=1$ corresponds to population size S_0 , and $G=0$ corresponds to infinite population size. (B) The information from A, presented in terms of population size rather than probability. The extrapolation of the GPE model to infinite population size gives an estimate of maximum speed, shown by the red dot.

fact, f has varied through time, and the potential effects of variation in f are addressed in the Discussion.

I apply this approach to the historical records of running speeds for races in which speed is correlated with population size. From records of population size and annual maximum speed, I construct an exceedance distribution of annual maximum speeds as described by Eqn 6, for which I then calculate the best fit values of ε , u , and σ using a least-squares criterion of fit. (Note that in this analysis, the value of the threshold u is chosen to give the best fit to the GPE rather than being chosen *a priori*.) The 95% confidence limits, for both individual parameters and the overall distribution, are calculated using 2000 iterations of a bootstrap sampling of the data with accelerated bias correction (Efron and Tibshirani, 1993). If $\varepsilon \leq 0$, the annual maximum speed estimated for an infinite population ($G=0$) provides an estimate of absolute maximum speed (Eqn 5), similar in principle (although not necessarily in magnitude) to the maximum speed estimated from the logistic equation (method 2).

To account for random variation about this best-fit population-driven model, deviations in speed from the best-fit GPE model are analyzed using Eqn 1. If the best-fit a for this distribution is <0 , an absolute maximum deviation exists (Eqn 2), and this maximum deviation is added to the expected value for the population-driven model.

The similarity between the population-based model of maximum speed and the time-based logistic model is highlighted in Fig. 2B where the information of Fig. 2A has been replotted to show how modeled speed (the red line) varies as a function of population size.

As an alternative to the approach used here, the effect of population size on running speed could be addressed by incorporating population size as a covariate in a GEV analysis similar to that used in method 1 (see Coles, 2001; Katz et al., 2005). For many human races, speed increases approximately linearly with the logarithm of population size. Using log population size as a linear covariate produces results essentially similar to those obtained with method 1 described above. However, this alternative method has not been fully explored, and its results are not reported here.

Population size

To implement the population-driven model, information is required regarding the year-by-year population of potential contestants. The combined number of thoroughbred foals born each year in the United States, Canada and Puerto Rico has been recorded by the US Jockey Club (www.jockeyclub.com/factbook08/foalcrop-nabd.html), and I assume that this represents a good approximation of the potential population from which Triple Crown runners are chosen. Horses racing the Triple Crown are 2 year olds, so the foal crop from a given year represents the potential racing population of the following year.

A request to the English Stud Book for records of racing greyhounds born in the UK each year was not answered, so an estimate of the trend in the potential population of racing greyhounds was obtained from records of the Irish Stud Book. This substitution was deemed acceptable for two reasons. First, many (perhaps most) greyhounds racing in Britain are born in Ireland. Second, Eqn 6 uses the ratio of initial population size to that in a given year. Thus, as long as the number of Irish greyhound births is proportional to that in the UK, the use of the Irish data is valid. Greyhounds begin racing at age 18 months to 2 years; I assume here that the number of dogs registered in a given year is approximately the number available to race in the following year.

Estimates of the world's human population were garnered from Cohen (Cohen, 1995) and the US Census Bureau (<http://www.census.gov/ipc/www/idb/worldpop.html>; 'Total midyear population for the world: 1950–2050'; accessed June, 2008). Population size N for a given year from 1850 to 2008 was estimated as:

$$N = -6.423X^5 + 40.144X^4 - 93.372X^3 + 102.39X^2 - 52.147X + 11.054, \quad (7)$$

where N is measured in billions and X is centuries since 1800 ($r^2 > 0.999$). I assume a 1:1 gender ratio for humans; the potential runners' population for men and women is thus each half the total world population. Note that this equation yields spurious values if used outside the years 1850–2008.

RESULTS

Thoroughbreds

Temporal patterns of winning speeds for the US Triple Crown are shown in Fig. 3. There is no significant correlation between year and winning speed in the Kentucky Derby for the period 1949 to

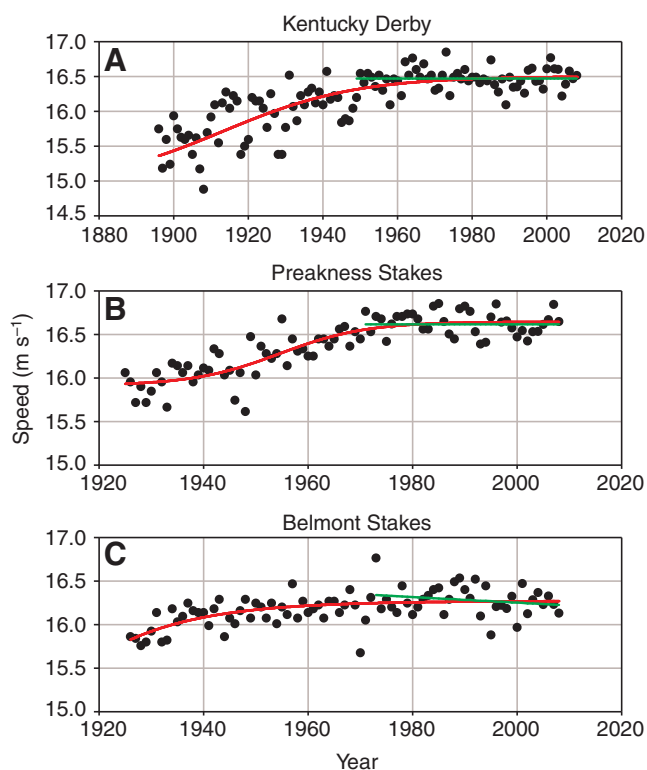


Fig. 3. Temporal patterns of winning speeds in the Triple Crown races. Black dots are winning speeds in the years shown. Green lines are regressions for data in the plateau of each record; any slope of these regression lines is statistically insignificant. Red lines are the best-fit logistic models.

2008. An apparent plateau was reached later in the Preakness Stakes (1971) and Belmont Stakes (1973). Significance levels for the correlation of speed with time in these plateau years are given in Appendix 3, Table A1.

Extreme-value analysis of race speeds during each plateau suggests that there is an absolute upper limit to speeds in each of these races (a is significantly less than 0 in each case), and the predictions are shown in Table 1 (no-trend model).

The temporal pattern of speed for each race is closely modeled by the logistic equation (Appendix 2 and Table A2) and, in each case, horses appear to have reached a plateau in speed (Fig. 3). Predictions of maximum speed made using a logistic model fitted to the entire data set for each race (Table 1, logistic model) are statistically indistinguishable from those obtained from the subset of plateau data.

For each race, the predicted absolute maximum running speed (averaged across methods) is only slightly (0.52% to 1.05%) faster than the current record.

Table 1. Predicted and current record maximum speeds for thoroughbreds running a distance of 1911–2414 m

	Plateau year	Predicted maximum speed (m s^{-1})			Average increase (%)
		Logistic model	No-trend model	Current record	
Kentucky Derby	1949	17.071	16.966	16.842	1.05
Preakness Stakes	1971	17.090	16.914	16.853	0.88
Belmont Stakes	1973	16.899	17.031	16.877	0.52

During the current plateau in speeds, there is no significant correlation between speed and year.

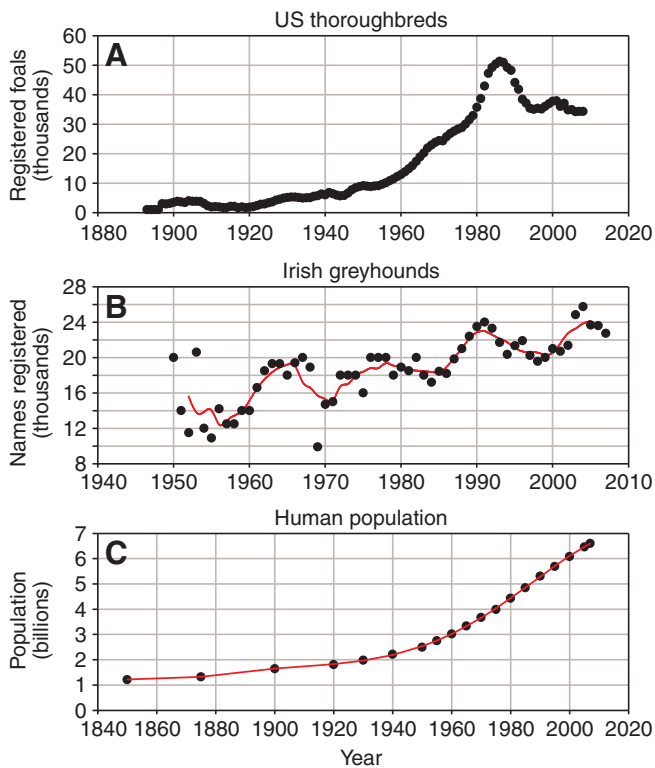


Fig. 4. Population trends in (A) US thoroughbreds, (B) Irish greyhounds and (C) humans. The red line in B is a 5 year running average of the data to emphasize the trend. The red line in C is from Eqn 7.

The potential population of Triple Crown runners increased dramatically from the 1880s until the mid-1980s, but has decreased since (Fig. 4A). Plots of speed as a function of population size (e.g. Fig. 5A) demonstrate that changes in population size are not the controlling factor in winning speeds in these horse races. Speed in the Kentucky Derby is not correlated with population size when the population is above 8400 ($P > 0.882$, Fig. 5A), in the Preakness Stakes when the population is greater than 24,300 ($P > 0.867$), and in the Belmont Stakes when the population is greater than 25,700 ($P > 0.253$). Because of the lack of correlation between population size and speed above certain population limits, the population-driven model of speeds was not applied to horses.

Greyhounds

In a pattern similar to that seen with horses, race speeds for greyhounds appear to have plateaued (Fig. 6). The plateau in the English Oaks began in approximately 1966 and in the English Grand National and English Derby in approximately 1971. The significance levels of the regression of speed on time during these plateaus are given in Appendix 3 (Table A3). The temporal pattern of speeds for each race is closely modeled by the logistic equation (details are given in Appendix 3, Table A4), and predicted maximum speeds calculated using this method (Table 2, logistic model) are very similar to those calculated from the plateaus alone.

Averaged across methods, predictions of maximum running speed for each race are only 0.29% to 0.92% higher than existing records (Table 2).

The estimated population of racing dogs increased gradually from 1950 (the earliest year in which records are available) to 2007 (Fig. 4B), with substantial year-to-year variation. Speed in the

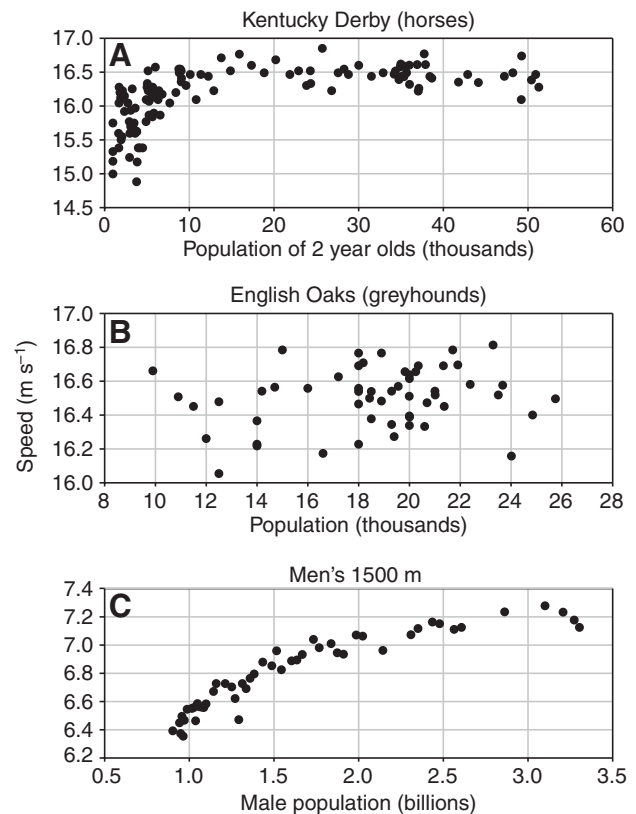


Fig. 5. Representative examples of variation in running speeds as a function of population size. For sufficiently large populations, there is no correlation between population size and speed in (A) thoroughbreds and (B) greyhounds. In contrast, human speeds (exemplified here by C, the men's 1500 m race) are correlated with population size throughout the historical range of population size.

English Oaks is not correlated with population size when the population is above 14,000 ($P > 0.354$), in the English Grand National when the population is greater than 19,300 ($P > 0.332$), and in the English Derby when the population is greater than 19,565 ($P > 0.232$). A representative example of the relationship between Irish greyhound population size and speed is shown in Fig. 5B. The fact that race speeds apparently plateaued while the population increased suggests that population size is not a substantial factor in the control of maximal speed in greyhounds, and the population-driven method of analysis was not applied to dogs.

Humans

Temporal patterns of human running speed are shown in Figs 7–9. For women running 100 m to 1500 m, speeds appear to have plateaued during the 1970s. Approximate onset years for each plateau and the corresponding probability level are given in Appendix 3 (Table A5). For these races, I applied extreme-value analysis directly to the data in each plateau. In the 200 m and 800 m races, an absolute maximum speed could be calculated (a was significantly < 0 , Table A5). In the 100 m, 400 m and 1500 m races, no absolute limit is defined for women's speeds; 100 year return values are given here (Table 3) and absolute maxima (if they exist) will be somewhat higher.

Data for all human races could be accurately fitted with a logistic model (for details see Appendix 3, Table A6). Results from the logistic models of human running speed are given in Table 3. In all

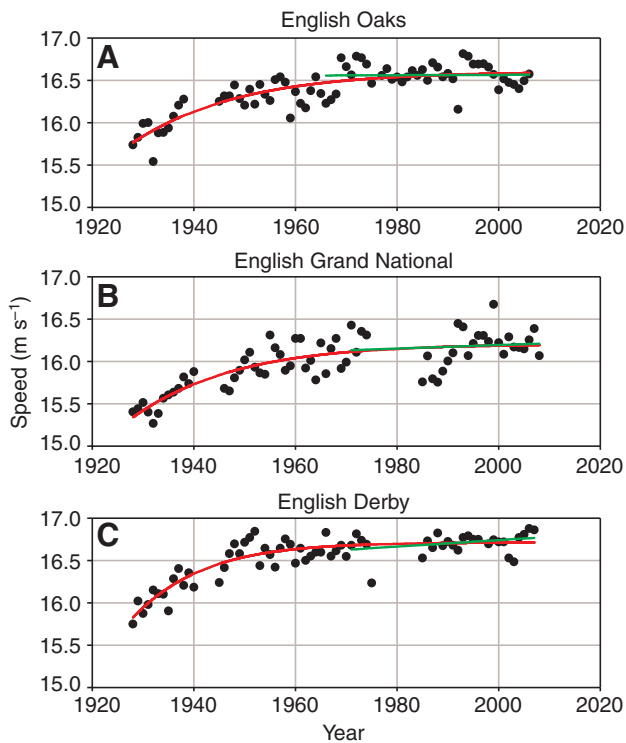


Fig. 6. Temporal patterns of winning speeds in English greyhound races. Black dots are winning speeds in the years shown. Green lines are regressions for data in the plateau of each record; any slope of these regression lines is statistically insignificant. Red lines are the best-fit logistic models. Gaps in the 1970s and 1980s for the English Grand National and English Derby are due to changes in the course length in these races during that period.

cases except the women's 400 m and 1500 m races, a in the GEV fit was <0 , and absolute maximum speeds could be predicted (Appendix 3, Table A6). For the women's 400 m and 1500 m races, values for 100 year return times are used. For the races in which speeds appear to have plateaued, predictions made using the logistic equation and the entire historical record are slightly higher than those from the analysis of the plateaus alone (Table 3).

The human population has exploded over the last century (Fig. 4C). In those races in which women's speeds have reached a plateau in recent years, a similar plateau is present in the relationship between speed and population (see Appendix 3, Table A7), and therefore it is unlikely that population is driving speed in these races. In all non-plateau races, however, a plot of speed *versus* population size shows a correlation throughout the record (a representative example is shown in Fig. 5C), and the population-driven model was applied. Due to large year-to-year variation in speeds recorded early in the twentieth century, the GPE fitted to data from men's 100 m

and 200 m races had exceptionally large confidence intervals (e.g. the 95% confidence interval for predicted absolute maximum speed included 0 m s^{-1}), and these questionable results are not included here. In the other races analyzed, the GPE provided an acceptable fit. Results from a representative example are shown in Fig. 10, and all results from this model are given in Table 3. In all non-plateau races, a was <0 (Appendix 3, Table A8), and an absolute maximum deviation from the fitted trend was calculated. Estimates from the population-driven model for non-plateau races closely match those obtained from other analytical approaches (Table 3), suggesting that increasing human population size will not be a major factor in future track records.

The results from all human races are summarized in Fig. 11 and Table 4. Speeds for which 100 year maxima are used (rather than absolute maxima) are shown as open symbols. Average predicted maximum speeds for men and women are only modestly faster than current world records (1.06% to 5.09% for men, 0.36% to 2.38% for women). The predicted potential for an increase in speed is not significantly correlated with race distance for men ($P > 0.93$). There is a marginally significant negative correlation between the potential for increase and race distance in women ($P = 0.037$), but this correlation is driven solely by the low predicted increase in speed in the marathon. Predicted maximum speeds for women are 9.3% to 13.4% slower than those for men, and in all but one instance (the 400 m race) the predicted maximum speed for women (including the confidence intervals) is less than the current record speed for men. There is a significant difference in the mean scope for increase (predicted maximum speed divided by current world record speed, minus 1) between men and women. For data pooled across all speeds, the mean scope for increase in predicted speed is 3.17% for men and 1.55% for women (Student's t -test, unequal variances, d.f.=12, $P = 0.008$).

DISCUSSION

These results provide tentative answers to the questions posed in the Introduction. For greyhounds, thoroughbreds and humans, there appear to be definable limits to the speed at which they can cover a given distance, and current record speeds approach these predicted limits. If present-day dogs, horses and humans are indeed near their locomotory limits, these animals (and the limits they approach) can serve as appropriate standards against which to compare predictions from mechanics and physiology.

The case for defined limits in horses and dogs is particularly strong. Despite intensive programs to breed faster thoroughbreds and greyhounds, despite increasing populations from which to choose exceptional individuals, and despite the use of any undetected performance-enhancing drugs, race speeds in these animals have not increased in the last 40–60 years. Thus, for horses and dogs, a limit appears to have been reached, subject only to a slight (and bounded) further increase due to random sampling. The situation is less clear cut for humans, in particular for men. Logistic and

Table 2. Predicted and current record maximum speeds for dogs running a distance of 460–480 m

	Plateau year	Predicted maximum speed (m s^{-1})			Average increase (%)
		Logistic model	No-trend model	Current record	
English Derby	1971	17.056	17.048	17.003	0.29
English Grand National	1971	16.826	16.826	16.673	0.92
English Oaks	1966	16.958	16.838	16.813	0.51

During the current plateau in speeds, there is no significant correlation between speed and year.

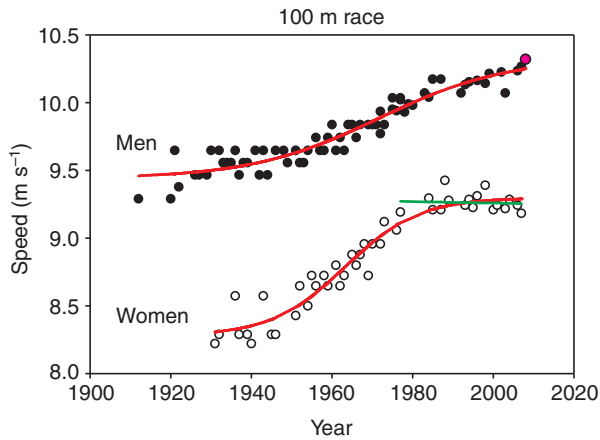


Fig. 7. Temporal patterns of annual fastest speeds for humans running 100 m. Dots are winning speeds in the years shown. The green line is the regression for data in the plateau of the women's record. Red lines are the best-fit logistic models. Men's speeds appear not to have plateaued. The recent world record set by Usain Bolt (2008 Olympics) is shown as the pink dot.

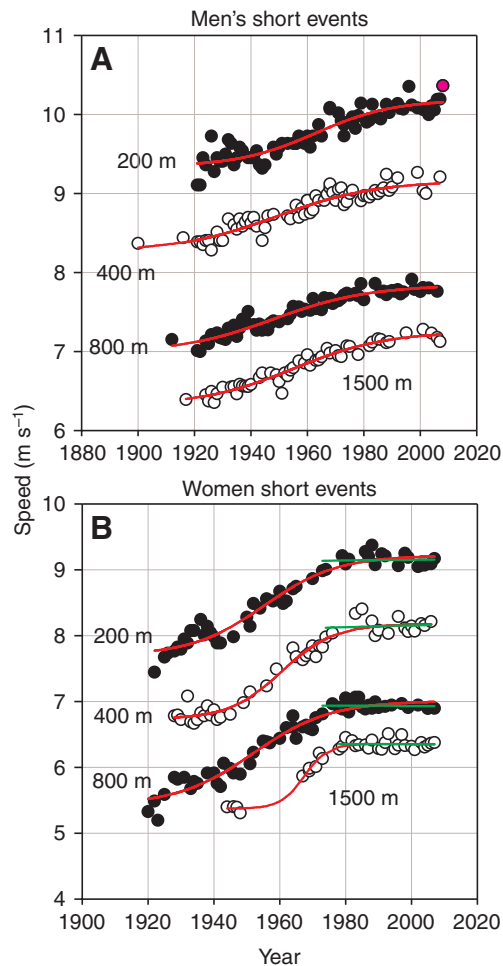


Fig. 8. Temporal patterns of annual fastest speeds for humans running 200 m to 1500 m; (A) men, (B) women. Dots are winning speeds in the years shown. Women's speeds appear to have plateaued, and the green lines are the regressions for data in these plateaus. Any slope of these regression lines is statistically insignificant. Red lines are the best-fit logistic models. Men's speeds appear not to have plateaued. The recent world record set by Usain Bolt in the 200 m race is shown as the pink dot.

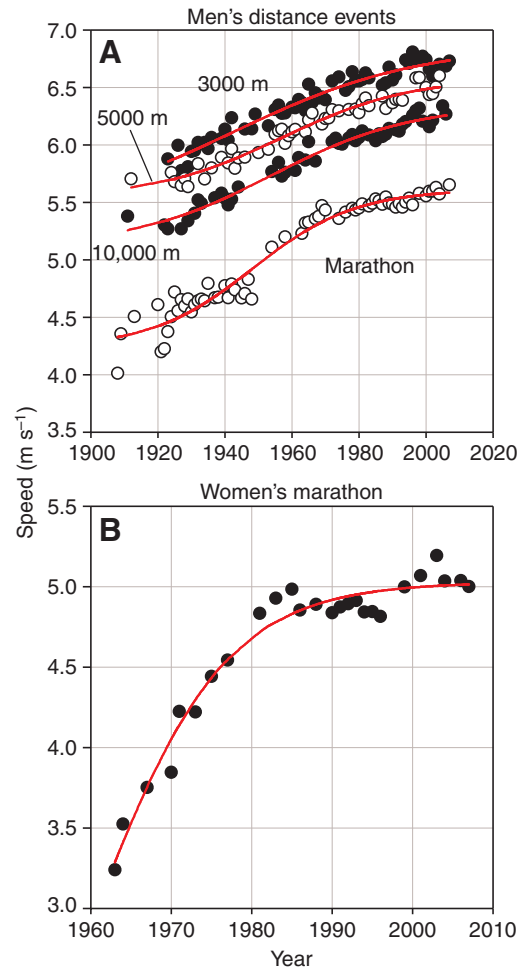


Fig. 9. Temporal patterns of annual fastest speeds for humans running 3000 m to 41,195 m (the marathon); (A) men, (B) women. Dots are winning speeds in the years shown. Red lines are the best-fit logistic models. Speeds in these distance races appear not to have plateaued.

population-driven models of the historical data suggest that a limit to male human speed exists, and that this speed is only a few per cent greater than that observed to date. But unlike speeds in horses and dogs, and sprint speeds for women, speeds for men have not yet reached a plateau.

An excellent example of the potential for a continued increase in men's speeds is provided by the recent world records set in the 100 m and 200 m races by Usain Bolt of Jamaica. Over a span of 3 days in the Olympic games of 2008, Bolt 'shattered' the then existing records, lowering the record in the 100 m from 9.72 to 9.69 s and in the 200 m from 19.32 to 19.30 s. Because Bolt is exceptionally tall for a sprinter (6'5", 1.96 m), he was hailed by the press as a physical 'freak' and the harbinger of a new era of sprinting.

Should Bolt's records cast doubt on the predictions made here? The answer is no. Bolt's records are only small improvements on the existing records for the 100 m and 200 m races, 0.3% and 0.1%, respectively, and Bolt's records are not out of line with the logistic fit to the historical data (Figs 7 and 8, pink dots). Furthermore, there have previously been similar jumps in record speed. Thus, as admirable as they are, there is nothing in Bolt's records to suggest that the predictions made here are inaccurate or that human speeds in the 100 m and 200 m races are limitless.

Table 3. Predicted and current record maximum speeds for human beings running races of various distance

	Distance (m)	Plateau year	Predicted maximum speed (m s^{-1})			Current record (m s^{-1})
			Logistic model	No-trend model	Population-driven model	
Men	100		10.55			10.32
	200		10.73			10.35
	400		9.42		9.31	9.26
	800		8.41		7.93	7.91
	1500		7.36		7.53	7.28
	3000		7.07		7.00	6.81
	5000		6.86		7.02	6.60
	10,000		6.64		6.66	6.34
	42,195		5.88		5.77	5.67
Women	100	1977	9.81	9.49		9.43
	200	1973	9.67	9.40		9.37
	400	1974	8.54	8.55		8.40
	800	1973	7.31	7.10		7.06
	1500	1978	6.55	6.61		6.51
	3000					6.17
	5000					5.84
	10,000					5.64
	42,195		5.30		5.12	5.19

Maximum speed predicted for the women's marathon using the population-driven method is slightly slower than the current world record speed, but the current world record is within the confidence limits of the estimate (see Fig. 11).

For distances of 100 m to 1500 m, women's speeds appear to have plateaued (Figs 7 and 8), superficially giving added confidence in the logistic model of the data. These plateaus (and this confidence) should be viewed with some skepticism, however. In each of these races, the current world record was set in the early to mid-1980s, a time when performance-enhancing drugs were becoming prevalent in women track athletes but before reliable mechanisms were in place to detect these drugs (Holden, 2004; Vogel, 2004). If speeds were artificially high in the 1980s due to drug use, and drug use was absent in subsequent years, one might suspect that the apparent plateau in the historical record could be an artifact. However, removing the annual maximum speeds from the 1980s does not substantially alter the results of the logistic analyses. Thus, the temporal plateaus in speeds at these distances appear to be real. There is an interesting corollary to this conclusion: if performance-enhancing drugs are still being used by women, the effect of the drugs has itself reached an apparent plateau.

In contrast to times in the shorter races, women's speeds in the marathon have continued to increase in recent years; like men, women in this race have not reached a demonstrable plateau. In this case, however, the current world record (5.19 m s^{-1} set by Paula Radcliffe in 2005) is very close to the average predicted absolute maximum speed (5.21 m s^{-1}); indeed, the current world record exceeds the maximum predicted from the population-driven model (although it lies within the confidence interval of this estimate). Given the upward trend in recent marathon speeds and the small difference between the current record and the predicted limit, this race is likely to provide the first test of the methods and predictions used here.

Maximum speeds predicted here are on average 1.63% higher than those predicted by Nevill and Whyte (Nevill and Whyte, 2005). The difference is probably due to the fact that the implementation of the logistic method here takes into account sampling variation in the maximum speeds.

My results for humans bolster the conclusion reached by Sparling and colleagues (Sparling et al., 1998) and Holden (Holden, 2004) that the present gender gap between men and women will never be closed for race distances between 100 m and the marathon. Note

that none of the data presented here speak to the possibility that women may someday out-run men at longer distances (Bam et al.,

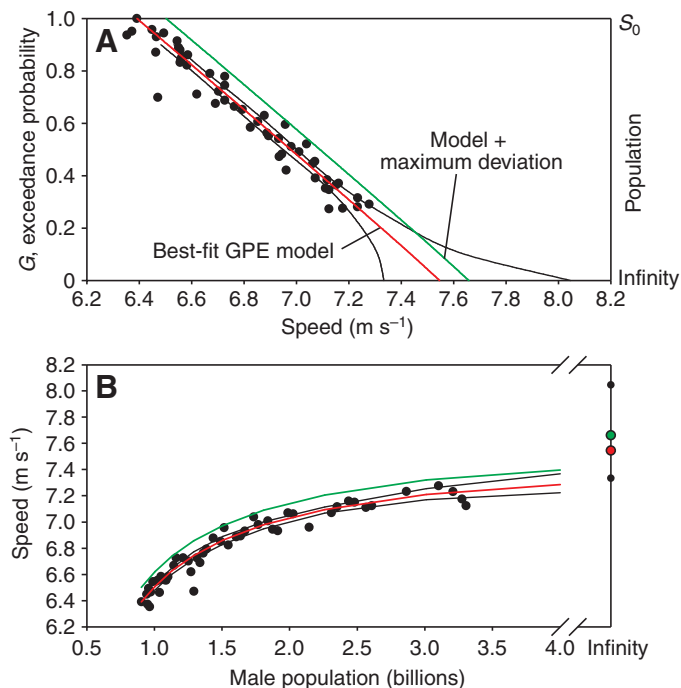


Fig. 10. (A) A representative example of analysis using the population-driven model; data for the men's 1500 m race. The red line is the best-fit GPE (Eqn 6), and the black lines are the bootstrap 95% confidence limits on this model. The green line depicts the addition of the absolute maximum deviation to the best-fit GPE model. The intersection of the green line with the abscissa is the predicted absolute maximum speed. In this case, the best-fit GPE is nearly linear ($\epsilon = -1.024$, where $\epsilon = -1$ is precisely linear), but linearity is neither an assumption of the analysis nor a necessity of the fit. (B) Data from A translated to show speed as a function of population size. Line colors are as in A. Dots on the ordinate at infinite population size show the estimated maxima of the various lines.

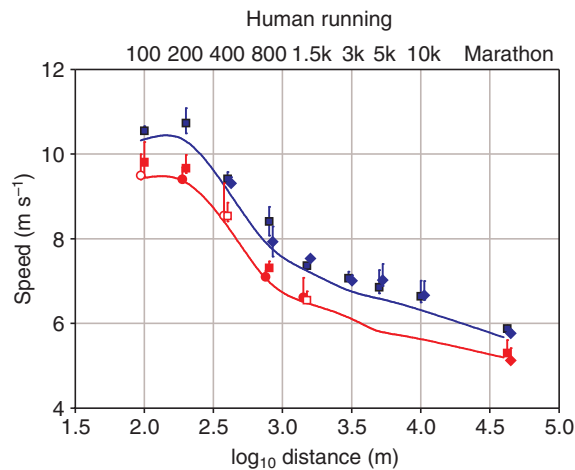


Fig. 11. Summary of human running data for distances 100 m to 42,195 m. Solid lines are the current world records for men (blue) and women (red). Estimates from the no-trend approach are shown by the circles; estimates from the logistic approach are shown by the squares; and estimates from the population-driven approach are shown by the diamonds. Error bars show the 95% range of the extreme-value estimate of absolute deviations. Estimates based on 100-year return values (rather than absolute maxima) are denoted with open symbols. For clarity, symbols are staggered slightly along the abscissa.

1997). Similarly, my results are consistent with the conclusion reached by Yang (Yang, 1975) that increases in human population will have only a minor effect on speed.

Evolution

Is it reasonable to suppose that the evolution of speed in horses has reached its limits? In a restricted sense, the answer is yes. The equipment used in horse racing, and the surfaces of the tracks on which these races are contested, did not change appreciably during the years when speeds were increasing in the Triple Crown races, and they have not changed since. Nor were there any apparent breakthroughs in training or nutrition that led to the increases in speed in thoroughbreds in the first half of the twentieth century. It seems

likely, then, that the initial increase in speed in horses was due primarily to selective breeding. If this is true, evidence from the Triple Crown races suggests that the process of selective breeding of thoroughbreds (as practiced in the US) is incapable of producing a substantially faster horse: despite the efforts of the breeders, speeds are not increasing, and current attempts to breed faster horses may instead be producing horses that are more fragile (Drape, 2008). The fastest speed in two of the Triple Crown races was set in 1973 by the same horse, Secretariat, and he was initially credited with a speed equal to the record in the third race (the Preakness Stakes) as well. (The timer malfunctioned in that race, however, and Secretariat's subsequently established official speed is slightly slower.) Thus, Secretariat approached the predicted absolute maximum speeds in all three of his Triple Crown races and therefore may represent a good approximation of the ultimate individual thoroughbred in races 1.25–1.5 miles long.

In a larger sense, however, the equine data presented here are preliminary at best. It may well be possible that different criteria for selective breeding of horses could produce a faster animal. Thoroughbreds have been recognized as a separate breed since the 1700s, and regulation of the breed has constrained its gene pool: thoroughbreds are less genetically diverse than other breeds of horses (Cunningham et al., 2001). The breed is effectively a closed lineage descended from as few as 12–29 individuals (Cunningham et al., 2001; Hill et al., 2002), and 95% of the paternal lineages in present-day thoroughbreds can be traced to a single stallion, The Darley Arabian. Selective breeding starting with different equine stock could perhaps yield faster horses. In this sense, then, the results presented here do not necessarily address the question of the maximum speed for the species *Equus caballus*.

The same arguments apply to greyhounds. Greyhounds have been bred for speed since antiquity, and the results presented here show a clear plateau in the ability of current selective breeding to produce a faster greyhound. However, given the extraordinary malleability of canine morphology (which includes everything from Chihuahuas to Great Danes), it is quite possible that different breeding strategies (perhaps starting with a different breed of dog) could produce a faster *Canis familiaris*.

Table 4. Current and predicted human records for standard race distances

	Distance (m)	Current world record	Average predicted absolute world record	Current record speed (m s^{-1})	Average predicted absolute maximum speed (m s^{-1})	Average increase in speed (%)
Men	100	9.69 s	9.48 s	10.32	10.55	2.22
	200	19.30 s	18.63 s	10.35	10.73	3.68
	400	43.18 s	42.73 s	9.26	9.36	1.06
	800	1:41.11	1:38.04	7.91	8.17	3.22
	1500	3:26.00	3:21.42	7.28	7.45	2.29
	3000	7:20.67	7:06.42	6.81	7.04	3.34
	5000	12:37.35	12:00.8	6.60	6.94	5.09
	10,000	26:17.53	25:03.4	6.34	6.65	4.93
	42,195	2:03.59	2:00.47	5.67	5.83	2.72
Women	100	10.61 s*	10.39 s	9.43	9.65	2.38
	200	21.34 s	20.99 s	9.37	9.53	1.73
	400	47.60 s	46.75 s	8.40	8.54	1.65
	800	1:53.28	1:50.83	7.06	7.20	2.01
	1500	3:50.46	3:47.92	6.51	6.58	1.12
	3000	8:06.11		6.17		
	5000	14:16.63		5.84		
	10,000	29:31.78		5.64		
	42,195	2:15.25	2:14.97	5.19	5.21	0.36

*The official world record is 10.49 s, but there is compelling evidence that that race was wind aided (Pritchard and Pritchard, 1994).

Once again, the situation is less clear for humans. Unlike the apparent case in horses and dogs, human runners have recently benefited from substantial improvements in training, equipment and nutrition. In some cases (women's sprints), these benefits may have reached their own limits. But in other races, continued improvement in training, equipment and nutrition may well be contributing to the continued increase in race speeds. Because these effects are inextricably entwined with the historical race data, the predictions I make here may be biased. In essence, these predictions assume that historical trends in training, equipment and nutrition (whatever they are) will continue into the future. It is always dangerous to make such assumptions. Competitive swimming provides an example of this potential effect: improvement in the design of full-body swimsuits (a breakthrough not contemplated 10 years ago) contributed to a rash of recent world records, 25 in the Olympics of 2008 alone. Until we know more about the mechanisms of improvement in training, equipment and nutrition, and more about their actual role in the historical running record, the magnitude of their effects on future running speeds will remain uncertain, and the predictions made here must be used cautiously.

And then there is the subject of artificial performance enhancement, which inevitably leads to philosophical questions pertaining to the definition of absolute maximum speed. For example, how should we define 'male', 'female' and even 'human' for the purposes of this study? If a woman artificially enhances the concentration of testosterone in her body, a large number of changes accrue that make her physiologically more like a man and capable of higher speeds (e.g. Holden, 2004). In this altered state, she may exceed any limits that might exist for unaltered individuals. At what point should such a hormonally enhanced woman no longer count as a woman in the analysis of maximum female human speed? Stanisława Walasiewicz (Stella Walsh) provides an intriguing example. She was the preeminent female sprinter of the 1930s, posting times that were not matched until the 1950s. She married in 1956 and was subsequently shot and killed during a supermarket robbery, where she was an innocent bystander. An autopsy revealed that (unbeknownst to her) she was a hermaphrodite, possessing both ovaries and testes. Presumably the anabolic steroids produced by her testes contributed to her athletic success (Lawson, 1997), but had she not been murdered, we would never have known. Where should she be placed in the record books? Even more vexing questions await us in the future. For example, the potential exists to genetically engineer human athletes for enhanced performance (e.g. Vogel, 2004). At what point does a genetically altered person no longer count as human? (Similar questions can be raised regarding drugs and gender in greyhounds and horses.)

For scientific purposes, these sticky questions can in large part be circumvented through the use of arbitrary definitions. As long as one defines practical criteria for 'female', 'male' or 'human' in formulating a mechanical/physiological model of locomotion, the predictions of that model can be compared with an appropriate set of test organisms. For present purposes, let us define a greyhound, thoroughbred or human (male or female) as an individual performing without drug or genetic enhancement. If drugs have contributed to the winning speeds in the races used here, speeds in the absence of these drugs would presumably have been slower. Thus, if we could definitively account for the effect of drugs in the historical record, the estimated maximum speeds I predict for unadulterated animals would, if anything, be slower, and my estimates are in this sense conservatively fast.

A similar conclusion applies to possible variation in the fraction of the overall human population that participates in running races. Recall that the analysis of the effect of population size on maximum

speed assumed that f , a fixed (although unspecified) fraction of all men and women, runs each year. In reality, it seems likely that f has increased through time as awareness of the sport has spread and more prize money has been made available. In particular, the fraction of women competing in running races has probably increased in recent decades. However, if we could take the presumed increase in f into account in this analysis, the effect would be to reduce the predicted maximum speed. (See Appendix 2 for an explanation.) In this respect, my results are again conservatively fast.

Some cynics have suggested that the problem of artificial enhancement in sports should be 'solved' by simply making drug and genetic enhancement legal. If such enhancement is allowed, the question of maximum human running speed becomes much more difficult to answer. On the one hand, it seems likely that humans have been, and still are, clandestinely employing performance-enhancing drugs despite the ban on their use. If so, the efficacy of these drugs is questionable: e.g. women's speeds in sprint races appear to have plateaued. On the other hand, it is impossible to rule out the possibility that new drugs or genetic enhancement could do for running what full-body swim suits have done for swimming: provide the means for dramatic improvement. In that case, the maximum speeds estimated here could be low.

What limits speed?

The analysis presented here deals solely with the results of competitive races, not with the factors that caused a certain individual to win or lose. In this respect, my results are as unsatisfying as those of previous statistical analyses: they tell us that speed has limits, but not what accounts for these limits. Nonetheless, the pattern of estimated maximal speeds provides information of potential value to physiologists and biomechanicians. It seems unlikely that a single mechanical or physiological factor could account for the limit to speed at all distances. The height and mass of elite runners differs among race distances (Weyand and Davis, 2005) as does the ability of aerobic capacity to predict speeds (Weyand et al., 1994; Weyand et al., 1999). It is striking, then, that the predicted scope for increased speed in humans is similar across distances ranging from 100 m to 42,195 m (Fig. 11; Table 4). This distance-independent scope for increase suggests that some sort of higher order constraint may act on the suite of physiological and mechanical factors to limit speed.

Context

The likelihood that there are limits to speed should not diminish the awe with which we view the performance of dogs, horses and humans. For example, a woman running the estimated absolute fastest speed for 100 m would have beaten the world's fastest male in 1955, a feat that would have astounded contemporary spectators. The predicted maximum speed (5.83 ms^{-1}) for a man running a marathon (42.2 km) would have been fast enough to beat the great Emil Zatopek in his world's best 10 km race in 1954. Those in the stands watching that race could not have imagined someone besting Zatopek by 16 s, and then simply continuing at that winning pace for another 32.2 km.

APPENDIX 1

Notes on the historical records

Horses

Until 1991, Triple Crown races were timed by hand to the nearest 0.2 s; since then they have been timed electronically to the nearest 0.01 s. I have made no correction for the shift from hand to electronic timing.

Humans

In some women’s races, the earliest data are extremely variable, perhaps because so few women participated in the sport at that time. For the 100m race, women’s data prior to 1931 were not used. For the 400m race, data prior to 1928 were not used.

World’s best times by a individual later proven to be using performance-enhancing drugs were deleted from the record; in those years, the second-best time was used. Times posted by individuals of uncertain gender (as noted by the ATFS) were not used; second-best times were substituted. The current women’s 100m world record (10.49s) was set by Florence Griffith Joyner in 1984, but there is compelling evidence that that race was wind aided (Pritchard and Pritchard, 1994). I have replaced this world’s best for 1984 with a time of 10.61 s (Griffith Joyner’s second-best time for that year).

Until the 1970s, human races were timed by hand. Because of the slight delay in starting a watch (due to human response time) and the potential for an early stop as an official anticipates the finish, hand times are slightly shorter than corresponding electronic times by approximately 0.165 s. This difference is negligible for races of 400m and longer, but can be a substantial factor in 100m and 200m races. Here, I have added 0.165 s to hand-timed 100m and 200m times. For 2–3 years during the switch-over from hand to electronic timing, world’s best times were recorded for both methods, and both hand-timed results (plus 0.165 s) and electronic results have been included here.

A race of 220 yards is very similar in length to a race of 200m (220 yards=201.168 m). The ATFS has reconciled the two races by recording 220 yard times minus 0.1 s as equivalent to 200m times for both men and women, and I follow this convention here. Similarly, I accept 440 yard times minus 0.3 s as equivalent to 400m times.

APPENDIX 2

Relating population size to exceedance probability

The generalized Pareto equation (GPE) is traditionally expressed as a cumulative probability distribution:

$$P(V) = 1 - \left[1 + \epsilon \left(\frac{V - u}{\sigma} \right) \right]^{-1/\epsilon} \tag{A1}$$

where $P(V)$ is the probability that the maximum speed in a randomly chosen sample of size S_0 is $\leq V$ (Coles, 2001). For the present purposes, it is more convenient to work with $1 - P(V) = G(V)$, where $G(V)$ is the chance of getting a sample maximum speed $>V$:

$$G(V) = \left[1 + \epsilon \left(\frac{V - u}{\sigma} \right) \right]^{-1/\epsilon} \tag{A2}$$

How many samples of size S_0 would one have to take (on average) to get a maximum speed $>V$? From the laws of probability (Feller, 1968; Denny and Gaines, 2000) it can be shown that this exceedance number E is:

$$E = \frac{1}{G(V)} \tag{A3}$$

If each sample has S_0 individuals and we need E samples (on average) to get a maximum speed $>V$, the overall number of individuals we need to sample to exceed V is:

$$S = ES_0 = \frac{S_0}{G(V)} \tag{A4}$$

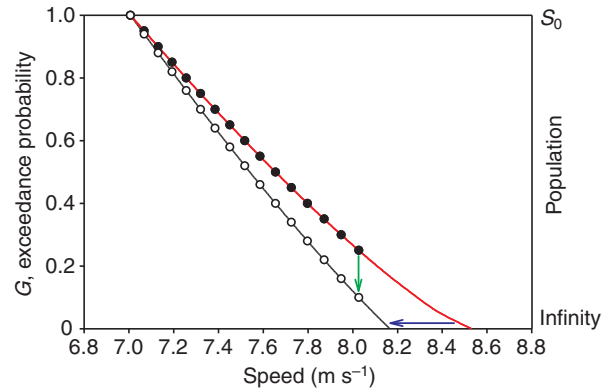


Fig. A1. If f , the fraction of individuals that run a particular race, increases through time, the actual population of runners associated with a given measured maximum speed (open dots) is higher than that supposed by the calculations made here (solid dots). A larger population corresponds to a smaller probability of exceedance (e.g. the green arrow). Thus, if probabilities were to be adjusted for an increase in f (open dots) the estimated absolute maximum speed would be reduced (the blue arrow).

Rearranging, we solve Eqn A4 for $G(V)$:

$$G(V) = \frac{S_0}{S} \tag{A5}$$

Upon inserting Eqn A5 into Eqn A2, we see that:

$$\frac{S_0}{S} = \left[1 + \epsilon \left(\frac{V - u}{\sigma} \right) \right]^{-1/\epsilon} \tag{A6}$$

If, in a given year, the world population of (for example) women is N and fraction f are runners, $S=fN$ women run the race that year. When measured in units of S_0 , the number of ‘samples’ of maximum speed we take in a given year is therefore S/S_0 . If f is constant across years, $S_0/S=N_0/N$ where N_0 is the world population in the year in which S_0 was measured. Thus:

$$G(V) = \frac{N_0}{N} = \left[1 + \epsilon \left(\frac{V - u}{\sigma} \right) \right]^{-1/\epsilon} \tag{A7}$$

In this fashion, N_0/N (and hence G) can be estimated from records of world population for each measured V . Plotting G as a function of V provides an estimate of the exceedance function described by Eqn 6 in the text. This is the recipe for estimating u , σ and ϵ discussed in the text.

It is possible that the fraction of the human population sampled by competition (especially the fraction of women) has increased over the last century. If so, maximum speeds predicted using the population-driven model are too high: estimates of cumulative probability incorporating an increasing fraction of runners would yield lower G values for recent years than those shown here, thereby lowering the estimates for extrapolated maximum speed (see Fig. A1).

APPENDIX 3

Details of the statistical results

Details of the statistical results are given in Tables A1 to A8.

Table A1. Parameters for the GEV (Eqn 1) for the plateau portion of the historical record for US Triple Crown races

	Plateau begins	Probability	<i>a</i>	<i>b</i>	<i>c</i>
Kentucky Derby	1949	>0.882	-0.2826	16.4077	0.1577
Preakness Stakes	1971	>0.867	-0.4390	16.5970	0.1392
Belmont Stakes	1973	>0.232	-0.2004	16.2169	0.1631

GEV, generalized extreme value; *a*, shape parameter; *b*, location parameter; and *c*, scale parameter (see Eqn 1). All values for *a* are significantly less than 0 ($P<0.05$). The probability cited here is the likelihood that any calculated correlation between speed and year is due to chance alone.

Table A2. Parameters of the logistic model for the US Triple Crown races

Race	mx	Low 95%	High 95%	mn	<i>k</i>	<i>t</i>	<i>a</i>	<i>b</i>	<i>c</i>
Kentucky Derby	16.498	16.387	16.609	14.925	0.056	1913.13	-0.4503	-0.0725	0.2906
Preakness Stakes	16.644	16.575	16.713	15.913	0.121	1954.51	-0.3288	-0.0520	0.1637
Belmont Stakes	16.281	16.230	16.333	3.145	0.053	1862.24	-0.2421	-0.0586	0.1640

The 95% confidence limits are given for mx, the estimated maximum of the logistic model. Values for mn (minimum fastest speed), *k* (shape parameter) and *t* (location parameter) of the logistic model are best-fit estimates. Values shown for *a*, *b* and *c* of the GEV (Eqn 1) are best-fit estimates. All values for *a* are significantly less than 0 ($P<0.05$).

Table A3. Parameters for the GEV (Eqn 1) for the plateau portion of the historical record for greyhound races

	Plateau begins	Probability	<i>a</i>	<i>b</i>	<i>c</i>
English Oaks	1971	>0.132	-0.5230	16.526	0.1631
English Grand National	1971	>0.606	-0.3023	16.105	0.2180
English Derby	1966	>0.922	-0.4165	16.662	0.1607

All values for *a* are significantly less than 0 ($P<0.05$). The probability cited here is the likelihood that any calculated correlation between speed and year is due to chance alone.

Table A4. Parameters of the logistic model for greyhound races

Race	mx	Low 95%	High 95%	mn	<i>k</i>	<i>t</i>	<i>a</i>	<i>b</i>	<i>c</i>
English Oaks	16.602	16.454	16.750	5.751	0.051	1879.21	-0.4734	-0.0468	0.1689
English Grand National	16.205	16.059	16.351	8.401	0.055	1889.88	-0.2597	-0.0650	0.1791
English Derby	16.713	16.636	16.789	8.489	0.079	1901.25	-0.3630	-0.0426	0.1402

The 95% confidence limits are given for mx, the estimated maximum of the logistic model. Values for mn, *k* and *t* of the logistic model are best-fit estimates. Values shown for *a*, *b* and *c* of the GEV (Eqn 1) are best-fit estimates. All values for *a* are significantly less than 0 ($P<0.05$).

Table A5. Parameters for the GEV (Eqn 1) for the plateau portion of the historical record for women's races

Distance (m)	Plateau begins	Probability	<i>a</i>	<i>b</i>	<i>c</i>
100	1977	>0.801	0.1506	9.2278	0.0400
200	1973	>0.846	-0.1516	9.1046	0.0901
400	1974	>0.695	0.0293	8.0870	0.0935
800	1973	>0.869	-0.1541	6.9078	0.0580
1500	1978	>0.987	0.1495	6.3156	0.0450

For the 200 m and 800 m races, all values for *a* are significantly less than 0 ($P<0.05$); *a* is significantly >0 for the 100 m and 1500 m races and indistinguishable from 0 for the 400 m race. The probability cited here is the likelihood that any calculated correlation between speed and year is due to chance alone.

Table A6. Parameters of the logistic model for various human races

	Distance (m)	mx	Low 95%	High 95%	mn	<i>k</i>	<i>t</i>	<i>a</i>	<i>b</i>	<i>c</i>
Men	100	10.330	10.165	10.495	9.439	0.063	1971.65	-0.3156	-0.0256	0.0771
	200	10.186	10.050	10.322	9.340	0.073	1962.95	-0.1859	-0.0486	0.1107
	400	9.160	9.035	9.285	8.262	0.057	1949.31	-0.3090	-0.0298	0.0884
	800	7.839	7.758	7.920	6.977	0.059	1947.94	-0.0940	-0.0274	0.0053
	1500	7.251	7.147	7.356	6.313	0.061	1955.12	-0.5325	-0.0141	0.0676
	3000	6.868	6.610	7.127	5.353	0.036	1942.45	-0.2803	-0.0221	0.0619
	5000	6.589	6.454	6.724	5.533	0.052	1956.90	-0.1796	-0.0226	0.0519
Women	10,000	6.350	6.208	6.492	5.114	0.048	1953.78	-0.1678	-0.0233	0.0528
	42,195	5.612	5.518	5.706	4.234	0.065	1948.28	-0.3599	-0.0315	0.1070
	100	9.297	9.222	9.373	8.277	0.109	1963.27	-0.1568	-0.0378	0.0858
	200	9.214	9.112	9.317	7.699	0.090	1955.42	-0.2468	-0.0457	0.1227
	400	8.174	8.097	8.251	6.732	0.127	1960.64	0.0303	-0.0512	0.0838
	800	7.006	6.909	7.103	5.425	0.086	1952.14	-0.3525	-0.0368	0.1200
	1500	6.355	6.325	6.385	5.372	0.277	1967.45	0.0020	-0.0281	0.0482
	42,195	5.028	4.974	5.082	0.775	1959.98	-0.3159	-0.0324	0.0967	

The 95% confidence limits are given for mx, the estimated maximum of the logistic model. Values for mn, *k* and *t* of the logistic model are best-fit estimates. Values shown for *a*, *b* and *c* of the GEV (Eqn 1) are best-fit estimates. With the exception of the 400 m and 1500 m races for women, all values for *a* are significantly less than 0 ($P<0.05$). For women's 400 m and 1500 m races, *a* is indistinguishable from 0.

Table A7. Correlation between population size and speed in women's races

Distance (m)	Plateau begins (population, billions)	Probability
100	2.393	>0.369
200	2.184	>0.146
400	2.063	>0.703
800	2.063	>0.334
1500	2.143	>0.987

The probability cited here is the likelihood that any calculated correlation between speed and population size is due to chance alone.

Table A8. Parameters of the population-driven model for various human races

	Distance (m)	Estimated			ϵ	σ	u	a	b	c
		model max.	Low 95%	High 95%						
Men	400	9.31	9.13	9.71	-1.314	1.358	8.273	-0.3140	-0.0299	0.0892
	800	7.93	7.85	8.08	-1.566	1.367	7.055	-0.0918	-0.0277	0.0560
	1500	7.53	7.32	8.08	-1.024	1.183	6.377	-0.5427	-0.0138	0.0679
	3000	7.00	6.82	7.31	-1.088	1.217	5.885	-0.2873	-0.0222	0.0633
	5000	7.02	6.70	7.84	-0.775	1.082	5.624	-0.1919	-0.0223	0.0522
	10,000	6.66	6.40	7.27	-0.933	1.303	5.264	-0.1850	-0.0229	0.0535
	42,195	5.77	5.65	5.99	-1.616	2.428	4.269	-0.3555	-0.0317	0.1058
Women	42,195	5.12	5.03	5.30	-4.266	7.948	3.259	-0.3334	-0.0325	0.1012

The 95% confidence limits are given for the estimated maximum of the model [$u-(\sigma/\epsilon)$]. Values for σ , ϵ and u of the GPE are best-fit estimates. Values shown for a , b and c of the GEV (Eqn 1) are best-fit estimates. All values for ϵ and a are significantly less than 0 ($P<0.05$).

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