

Review

The constructal law of organization in nature: tree-shaped flows and body size

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Summary

The constructal law is the statement that for a flow system to persist in time it must evolve in such a way that it provides easier access to its currents. This is the law of configuration generation, or the law of design. The theoretical developments reviewed in this article show that this law accounts for (i) architectures that maximize flow access (e.g. trees), (ii) features that impede flow (e.g. impermeable walls, insulation) and (iii) static organs that support flow structures. The proportionality between body heat loss and body size raised to the power $3/4$ is deduced from the discovery that the counterflow of two trees is the optimal configuration for achieving (i) and (ii) simultaneously: maximum fluid-flow access and minimum heat leak. Other allometric examples deduced from the

constructal law are the flying speeds of insects, birds and aeroplanes, the porosity and hair strand diameter of the fur coats of animals, and the existence of optimal organ sizes. Body size and configuration are intrinsic parts of the deduced configuration. They are results, not assumptions. The constructal law extends physics (thermodynamics) to cover the configuration, performance, global size and global internal flow volume of flow systems. The time evolution of such configurations can be described as survival by increasing performance, compactness and territory.

Key words: constructal law, tree, dendritic, body size, body heat, hair, fur, flight, allometric laws, organ size.

The broad view: biology, physics and engineering

The occurrence of flow configuration (shape, structure) is a phenomenon so universal that it unites the natural with the engineered, and the animate with the inanimate. From 1996, constructal theory has shown that flow architectures such as trees and round tubes can be deduced from a single law of maximization of access for currents (Bejan, 1996, 1997a, 2000). If this law is correct, then how do we account for the occurrence of configurations that obstruct the flow of currents? Hair and fur prevent the flow of heat from the body of the animal to the ambient surroundings. Pairs of blood vessels in counterflow serve a similar insulation function in the tissue under the skin. Insulation technologies are everywhere in engineering. Much more obvious examples come from the flow of fluids: impermeable walls are everywhere, in fact, without them there would be no 'streams'. How can we reconcile such obvious contradictions with the maximization of flow access?

Even if the configurations that maximize flow access could be put with those that obstruct flow under the same theoretical tent, it would cover only flow systems. So how can we account for the occurrence of refined mechanical (non-flow) structures such as the skeletons of animals and the frames of aeroplanes? Principles of maximum stress uniformity and minimum weight have been invoked to account for the generation of architecture

in mechanical support structures. How can such 'static' principles be reconciled with the principle that unifies flow systems?

In this paper I answer these questions by reviewing a series of recent developments based on constructal theory, where geometry is the big unknown — the mechanism by which the global performance of the flow system is maximized. The argument goes as follows: 'what' flows (fluid, heat, electricity, goods, people) is not as important as 'how' the flow derives its architecture from the competition between objectives and constraints. The maximization of 'access' means many things, depending on what flows; for example, the minimization of flow resistance and pumping power in fluid flow (from blood vessels to atmospheric circulation), the minimization of electrical resistance and Joule heating in all electrical networks (computers, power grids), and the minimization of travel time and cost in transportation and business (the Fermat principle of urban design and economics). In isolated thermodynamic systems, it means the acceleration of mixing *en route* to equilibrium and no flow.

In engineering, where the heat engine was the stimulus for the discovery of thermodynamics, the constructal law delivers precisely what Sadi Carnot called for: the minimization of

friction and shocks in fluid flow, and the avoidance of large temperature differences in heat flow. Such thermodynamic ‘imperfection’ cannot be avoided, because of size and time constraints. Resistances will always be present. As Poirier (2003) put it recently, the only way up on the ‘staircase to heaven’ envisioned by Sadi Carnot is by arranging and balancing the resistances against each other. To arrange and to distribute is to make the drawing, to deduce what was missing — the architecture. Optimal distribution of imperfection is the constructal law of geometry generation.

Natural flow systems exhibit the same tendency. The largest engine on earth — the wheels of atmospheric and oceanic circulation — achieves the same objective as birds, aeroplanes, and many other blobs of organized material movement (‘streams’) such as eddies of turbulence: to facilitate the movement of matter all over the globe, i.e. to maximize the mixing of the matter that is the globe. River basins are trees, in accordance with the constructal law of maximization of flow access. River cross-sections have a universal proportionality between width and depth, which can be deduced from the constructal law.

If animal design proceeds in accordance with the constructal law, then the animal destroys less exergy (i.e. useful energy, fuel) and requires less food. ‘Animal design’ means currents that flow along maximum-access paths between organs, currents that are guided by walls, not currents that leak directly to the ambient. Fuel or food management means that the engine and the animal must carry the optimal weight that makes the *whole* animal efficient, not the individual organ. From this comes the need to spread maximum stresses uniformly, and to support the flow structure with a mechanical structure having minimal weight. There is no contradiction between the constructal law of flow architecture and weight (size) as a constraint for the mechanical structure that supports the flow structure.

Along this theoretical route the constructal law provides the physics that is missing from the Darwinian principle of the fittest animal being the one that survives. It provides the physics definition of what is meant by ‘the fittest,’ or by its equivalent ‘the survivor,’ not only in biology but also in engineering, geophysics and economics, where selection and evolution are also evident. According to constructal theory therefore, animals, river basins and all of us — the ‘man + machine species’ — are the same.

Constructal law

The constructal law was first published in 1996 in the context of optimizing the access to flow between one point and an infinity of points (area or volume, in two- or three-dimensional systems, respectively), with application to traffic (Bejan, 1996), the cooling of small-scale electronics (Bejan, 1997b), and living fluid trees (Bejan, 1997c,d):

‘For a finite-size open system to persist in time (to survive) it must evolve in such a way that it provides easier and easier access to the currents that flow through it.’

The constructal law is about the time arrow of the phenomenon of flow architecture generation. It is a self-standing law, distinct from the second law of thermodynamics (see the concluding section of this article). Constructal theory has been reviewed in books (Bejan, 1997a, 2000, 2004; Rosa et al., 2004; Bejan et al., 2004) and in articles (Poirier, 2003; Lewins, 2003; Torre, 2004; Guerreri, 2004).

According to the constructal law, in the case of a flow between one point and an infinity of points, the flow path was constructed as a sequence of steps starting with the smallest building block, the size of which is fixed, and continued in time with larger building blocks (assemblies, constructs). The mode of transport with the highest resistivity (slow flow, diffusions, walking and high cost) was placed at the smallest scale, filling completely the smallest elements. Modes of transport with successively lower resistivities (fast flow, streams, vehicles, and low cost) were placed in the larger constructs, where they were used to connect the area-point or volume-point flows integrated over the constituents. The geometry of each building block was optimized for area-point access. The architecture that emerged was a tree in which every geometric detail was a result — the tree, as a geometric form *deduced* from a single principle.

A simple illustration of the discovery of the tree geometry is in the minimization of travel time between an infinity of points (area A) and one point (M , Fig. 1A). The deduction of the flow architecture from the constructal law is atomistic: from small to large, in time (see A_1, A_2, A_3, \dots ; Fig. 1A). There are at least two modes of locomotion: slow (walking, speed V_0), and fast (vehicles, $V_1 < V_2 < V_3, \dots$). The construction starts with the smallest elemental area, $A_1 = H_1 L_1$, where the A_1 size is fixed but the shape H_1/L_1 may vary. The elemental size A_1 is dictated by the land property and culture of those who live on A_1 .

The minimization of the travel time between all the points of A_1 and the boundary point M_1 generates four geometric features: (i) the V_1 street is placed on the longer axis of the $H_1 L_1$ rectangle; (ii) the slow movement covers A_1 entirely, i.e. every inhabitant of A_1 must first walk before reaching the street; (iii) the rectangle shape H_1/L_1 ; and (iv) the approach angle α_1 . For example, when $V_1 \gg V_0$, the optimal design of the elemental area is $H_1/L_1 = 2V_0/V_1$ and $\alpha_1 = 0$.

Interesting is that the optimal A_1 configuration can be deduced in two ways (Bejan, 1996, 1997a). One is the altruistic approach, in which the inhabitant with the worst geographical position is considered (point P, Fig. 1B–D), and the travel time from P to M_1 is minimized. The other is the egotistical approach, where the travel time of every inhabitant of the white area is calculated, averaged over A_1 , and then minimized. Both methods yield the same geometry. The architecture that is good for the most peripheral member of the assembly is good for the assembly as a whole: the urge to organize is an expression of selfish behavior. We return to this important aspect of the theory in the discussion of Fig. 6.

The opportunity to optimize geometry continues at

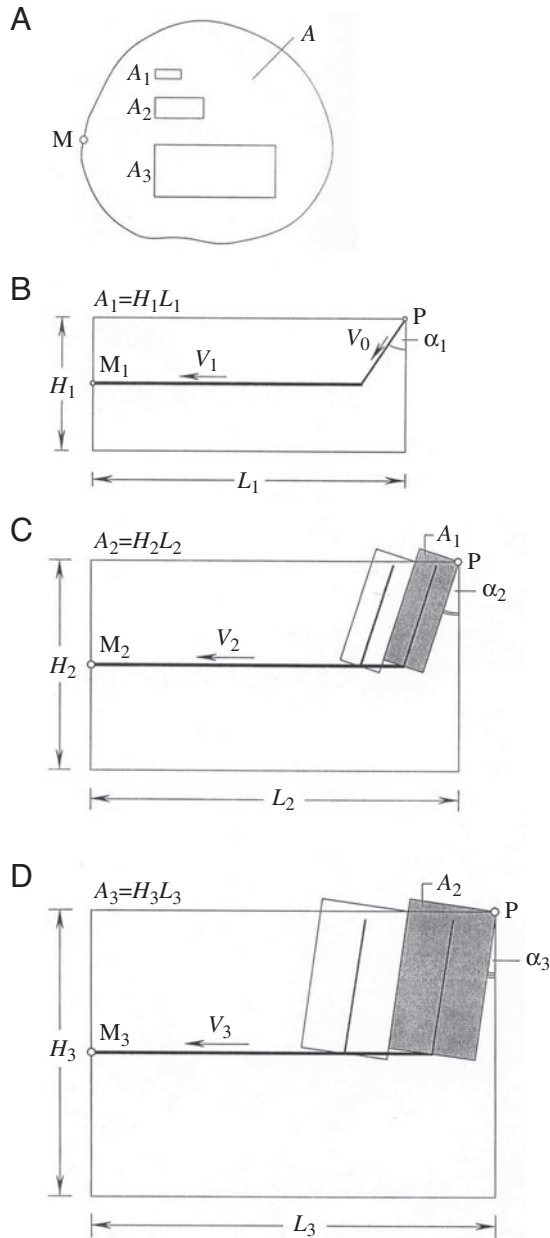


Fig. 1. (B–D) The constructal sequence for the minimal-time route between one point and the infinity of points A_1 , A_2 and A_3 of a finite-size area (A) (Bejan, 2000).

progressively larger scales. As shown in Fig. 1, a first assembly (A_2) consists of a number (n_2) of optimized elements (A_1). The new street of speed V_2 ($>V_1$) collects or distributes the traffic that covers the elements. The minimization of travel time between the area A_2 and the point M_2 calls for placing the V_2 street on the axis of A_2 , touching every point of A_2 with slow movement, and optimizing two geometric features, the shape H_2/L_2 (or n_2) and the approach angle α_2 .

The generation of geometry continues with assemblies of progressively higher order and larger size. Every feature of the emerging drawing is the result of invoking one principle: the maximization of access for traffic that connects one point with

the infinity of points that make up the area. Nothing is assumed, postulated or copied (modeled) from nature.

The optimized features of constructs of higher order (A_i , $i > 2$ in Fig. 1D) fall into a pattern that can be summarized as a simple algorithm. If these theoretical recurrence formulas were to be repeated *ad infinitum*, only then would the resulting image be a fractal (Bejan, 1997a; Avnir et al., 1998). The image of Fig. 1 is not a fractal. In other words, constructal optimization of volume-to-point accounts for why natural structures look like images generated by fractal algorithms truncated at a small but finite length scale, why such a cut-off scale exists, and why in a natural structure the algorithm breaks down in the steps situated close to the smallest scale (e.g. step $i=1$ in Fig. 1B).

Three-dimensional volume-to-point access has been optimized in the same manner, and the result is a tree architecture (for reviews, see Bejan, 1997a, 2000). In fluid trees, the structure is visible as channels and ducts if the flow possesses at least two regimes with dissimilar resistivities, high and low. The high-resistivity regime (e.g. viscous diffusion, Darcy flow) covers most of the space, as it fills the interstices formed between the smallest channels. The channels and streams are characterized by much lower resistivity (e.g. flow in ducts and streams). The structure is not visible — it is not even an issue — when only one flow regime (viscous diffusion) is present.

In the constructal fluid tree the dimension changes settle into a pattern (e.g. dichotomy) after the order of the volume construct becomes high enough. Dichotomy is not an assumption — it is an optimization result deduced from the constructal law (Bejan et al., 2000). Furthermore, the optimized diameter factor obeys Murray's law ($D_{i+1}/D_i = 2^{1/3}$) after the second construct (Murray's law was originally derived for fully developed laminar flow; the corresponding result for turbulent flow is discussed in Bejan et al., 2000). The step factor for tube lengths L_{i+1}/L_i exhibits a cyclical pattern for each sequence of three construct sizes, provided that $i > 2$. The theoretical tree has a definite (finite, known) beginning: the smallest scale and the optimized first construct. The geometry and finite size of this beginning distinguish this theoretical construction from the algorithms assumed and used in fractal geometry. The inner cut-off, and the breakdown of the algorithm at small-enough scales, are as important as any other geometric feature. The flow through the arms of the tree is as important as the invisible flow across the armpits.

The formulae that accompanied the fluid tree construction were the minimum necessary for making the drawing, and are tabulated in Bejan (1997a,c). As shown in Bejan (2000), more information can be obtained from the tables, for example, the total volume, the total internal surface of all the tubes (A), the volume-averaged porosity, the total mass (M_b) and the cross-sectional area of the tree root. Interesting relationships emerge when these quantities are plotted against each other, for example, the near-proportionality between A and M_b (see Eqn 9 below).

Fig. 2. The construction of the tree of convective heat currents: (A) the constrained optimization of the geometry of a T-shaped construct; (B) the stretched tree of optimized constructs; (C) the superposition of two identical trees oriented in counterflow, and (D) the convective heat flow along a pair of tubes in counterflow (Bejan, 2000). For a detailed explanation, see text.

Heat loss vs body size

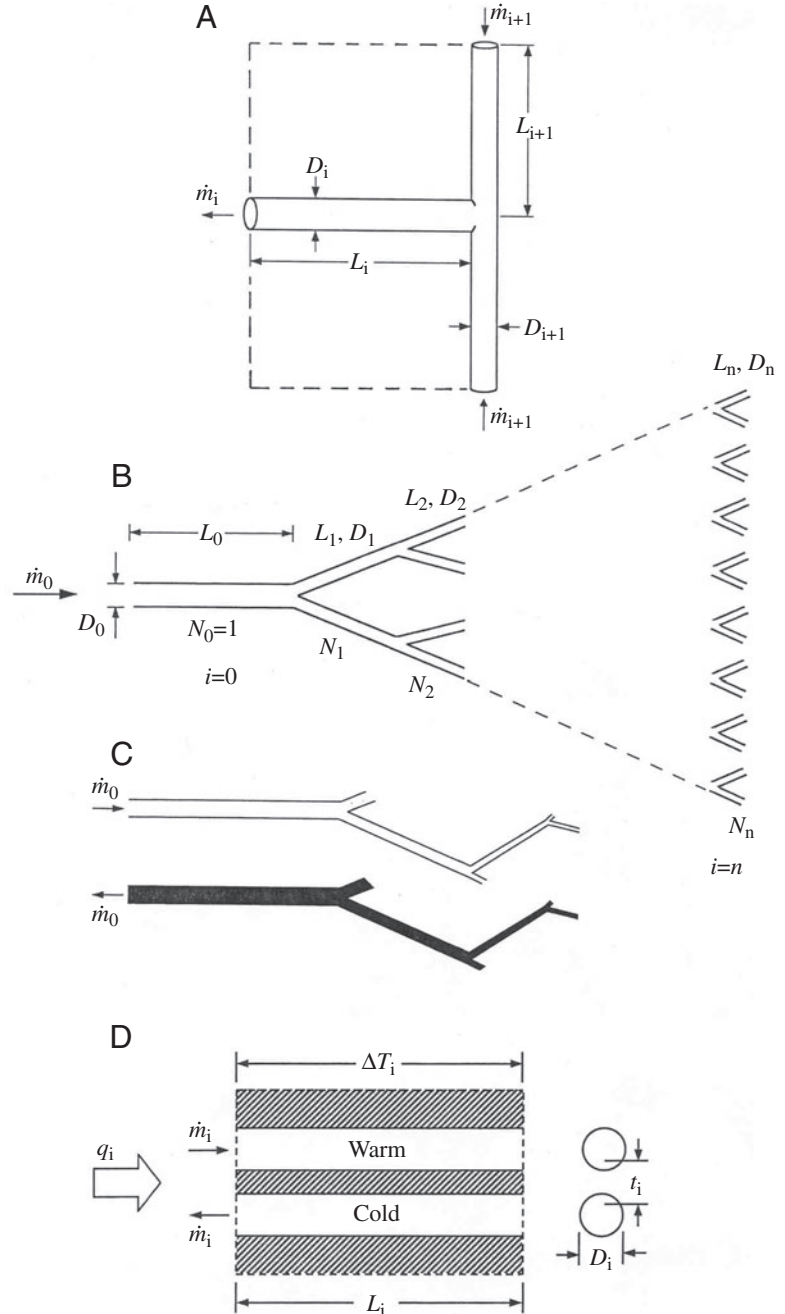
Constructal theory predicted the proportionality between metabolic rate and body mass raised to the power 3/4, by invoking the constructal law twice (cf. the introductory section of this article): in the minimization of body heat loss, and the minimization of blood pumping power (Bejan, 2000, 2001). The minimization of pumping power yields the constructal fluid tree (Bejan, 1997a,c): this can be derived more succinctly by optimizing a plane construct consisting of a plane T-shaped junction (Bejan et al., 2000; Fig. 2A). For simplicity, assume right angles and Hagen–Poiseuille flow with constant properties in every tube. The stream \dot{m}_i encounters the flow resistance of two L_{i+1} tubes in parallel, which are connected in series with one L_i tube. When the resistance is minimized by fixing the total tube volume, we find $D_{i+1}/D_i=2^{-1/3}$, which is independent of the tube lengths (L_i, L_{i+1}) and the relative position of the three tubes. Next, we optimize the lengths when the space allocated to the construct is fixed, $2L_{i+1}L_i=\text{constant}$. This yields the optimal ratio $L_{i+1}/L_i=f=2^{-1/3}$, where the smallest length scale is labeled $i=n$, and largest $i=0$.

The trees of blood vessels are an architectural feature under the skin, but not the only one. The other is the *superposition* of the arterial and venous trees, so closely and regularly that tube i of one tree is in counterflow with tube i of the other (Fig. 2C,D). This is a thermal insulation feature.

The arterial stream is warmer than the venous stream: heat flows transversally, from stream to stream. Because the enthalpy of the warmer stream is greater than that of the colder stream, the counterflow convects longitudinally the energy current $q_i=\dot{m}_i c_p \Delta T_{i,i}$, where c_p is the specific heat of blood, and $\Delta T_{i,i}$ is the stream-to-stream temperature difference at level i . It was first shown in heat transfer (Bejan, 1979) and later in bioengineering (Weinbaum and Jiji, 1985), that such a counterflow sustains a longitudinal temperature gradient, $\Delta T_i/L_i$, and that the convective energy current is proportional to this gradient:

$$q_i = \left[\frac{(\dot{m}_i c_p)^2}{h_i p_i} \right] \frac{\Delta T_i}{L_i}, \tag{1}$$

where h_i is the overall stream-to-stream heat transfer coefficient and p_i is the perimeter of contact between the two



streams. The stream-to-stream thermal resistance h_i^{-1} is the sum of two resistances: the resistance through the fluid in the duct ($\sim D_i/k_f$, where k_f is the fluid thermal conductivity), plus the resistance through the solid tissue that separates two tubes ($\sim t_i/k$, where D_i is diameter and k is the tissue thermal conductivity; t_i is defined in Fig. 2D: t_i is the average thickness of the tissue that separates two adjacent D_i tubes). Even when the tubes touch, t_i is of the same order as D_i . In addition, because $k_f \sim k$, we conclude that $h_i \sim k/D_i$, and Eqn 1 becomes:

$$\Delta T_i \sim \frac{q_i L_i k}{\dot{m}_i^2 c_p^2}. \tag{2}$$

The double-tree fluid structure is a single tree of convective heat leakage with zero net mass flow. The convective tree stretches from the core temperature of the animal (at $i=0$) to the skin temperature. The latter is registered in many of the elemental volumes ($i=n$) that are near the skin. The many counterflow pairs of the two fluid trees sustain the overall temperature difference ΔT :

$$\Delta T = \sum_{i=0}^n \Delta T_i \sim \frac{q_0 k}{\dot{m}_0^2 c_p^2} \sum_{i=0}^n N_i L_i. \quad (3)$$

In going from Eqn 2 to Eqn 3, we used the continuity relations for fluid flow ($N_i \dot{m}_i = \dot{m}_0$, constant) and heat flow ($N_i q_i = q_0$, constant). Recalling the L_{i+1}/L_i constant f , we substitute $L_i = L_0 f^i$, $L_n = L_0 f^n$ and $N_i = 2^i$ into Eqn 3:

$$q_0 \sim \left(\frac{q_0}{\dot{m}_0} \right)^2 \left\{ \frac{k L_n f^{-n} [(2f)^{n+1} - 1]}{c_p^2 \Delta T (2f - 1)} \right\}. \quad (4)$$

The right side has quantities that are constant, and quantities that depend on n (the number of construction steps). The ratio q_0/\dot{m}_0 is independent of body size (n) because both q_0 and \dot{m}_0 are proportional to the metabolic rate.

The volume inhabited by the tree is estimated by considering the stretched tree as a cone in Fig. 2B. The base of the cone (at $i=n$) has an area of size $N_n L_n^2 \sim 2^n L_n^2$. The height of the cone is of the same order as the sum of all the tube lengths, $L_0 + L_1 + \dots + L_n = L_0(1 - f^{n+1})/(1 - f)$, and the volume scale is:

$$V \sim L_n^3 \left(\frac{2}{f} \right)^2 \left(\frac{1 - f^{n+1}}{1 - f} \right). \quad (5)$$

The relationship between metabolic rate and total volume is obtained by eliminating n between Eqns 4 and 5. The result is visible in closed form if n is sufficiently large so that $(2f)^{n+1} \gg 1$ in Eqn 4, and $f^{n+1} \ll 1$ in Eqn 5. In this limit q_0 is proportional to 2^n , and V to $(2/f)^n$. From this follows:

$$q_0 = (\text{constant}) V^{3/4}. \quad (6)$$

It can be verified numerically that Eqn 6 also holds for small n . In conclusion, the proportionality between metabolic rate and body size raised to the power 3/4 is predictable from pure theory.

Constructal theory also anticipates the proportionality between breathing (or heartbeating) time and body size raised to the power 1/4 (Bejan, 2000). In one of the first constructal papers (Bejan, 1997d), it was shown that the pumping power required by the heart for blood circulation and the thorax for breathing is minimal if (a) the flow is intermittent (in and out, on and off), and (b) the 'in' time interval (t_1) is of the same order of magnitude as the 'out' time interval (t_2). Features (a) and (b) come from pure theory (the constructal law), not from observations. The optimal time scale ($t_{1,2} \sim t$) is:

$$t \sim \left(\frac{AD^{1/2} \Delta C}{\dot{m}} \right)^2, \quad (7)$$

where A is the total internal contact area of all the tubes of the tree, D is the mass diffusivity, ΔC is the concentration difference that drives the mass transfer process, and \dot{m} is the total mass flow rate of the tree (blood, air). The flow rate \dot{m} is proportional to the metabolic rate of the animal. Eqn 7 shows that in order to predict t as a function of body mass (M_b) we need expressions for $\dot{m}(M_b)$ and $A(M_b)$. From the optimized tree of convective currents we obtained Eqn (6), or:

$$\dot{m} \sim M_b^{3/4}. \quad (8)$$

To predict the relationship $A(M_b)$, we argue that the thickness of the tissue penetrated by mass diffusion during the breathing or heartbeating time t is proportional to $t^{1/2}$. The body volume (or mass) of the tissue penetrated by mass diffusion during this time obeys the proportionality relationship $M_b \sim A t^{1/2}$. Eliminating t between $M_b \sim A t^{1/2}$ and $t \sim (A/\dot{m})^2$, and using Eqn 8, we conclude that the contact area should be almost proportional to the body mass:

$$A \sim M_b^{7/8}. \quad (9)$$

Finally, the proportionalities in Eqns 8 and 9 and Eqn 7 mean that:

$$t \sim M_b^{1/4}. \quad (10)$$

This allometric law is supported convincingly by the large volume of observations accumulated in the physiology literature (Schmidt-Nielsen, 1984).

The constructal law for body heat loss vs size, Eqn 6, stands out on the background provided by the history of the theoretical attempts to predict this relationship. The earliest was Rubner's heat transfer model: because the convective heat loss is proportional to the body surface, the metabolic rate must be proportional to the length scale ($V^{1/3}$) squared, i.e. the body mass or volume raised to power 2/3. The heat transfer model was discredited by more recent observations of birds and mammals, which suggest an exponent closer to 3/4 than 2/3. Because of such observations, heat transfer was not included as a feature in recent fluid mechanics tree network models (e.g. West et al., 1997), which, by the way, is a good illustration of why modeling is empiricism. We return to these models in the concluding section.

Why should anyone question the currently accepted models by resurrecting heat transfer? First, and this is key, modeling is not theory. Models are simplified descriptions (facsimiles) of objects observed in nature. Second, the minimization of pumping power, which is invoked by modelers, is the constructal law, because less pumping power means less exergy [useful (liberated) energy; see below] destruction, and less food for the animal to survive. But, a lower heat leak also means less food and less exergy destruction. This is why the minimization of pumping power goes hand-in-hand with the old heat-loss doctrine, not against it. Minimum pumping power consumption and minimum loss of body heat are parts of the same constructal law — how to be constructed to be the fittest (the survivor), how to perform best, how to flow best.

Additional support for the constructal theory of body heat

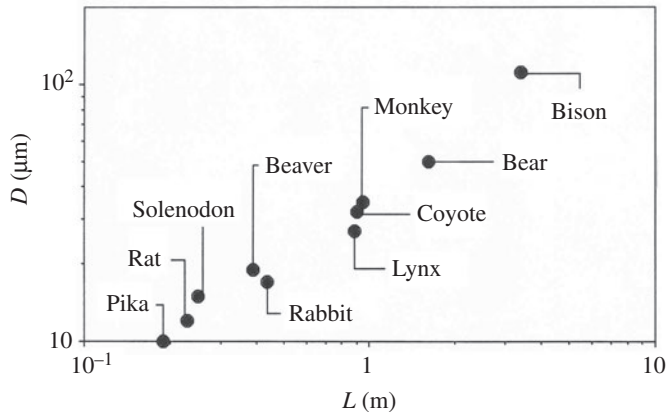


Fig. 3. The allometric law for animal hair strand diameter and body length scale.

loss comes from the allometric laws of the design of the hair coats of animals, such as the proportionality between the hair strand diameter and the animal body length scale raised to the power $1/2$ (Fig. 3). This allometric law was predicted (Bejan, 1990a,b) by minimizing the body heat loss through the hair coat, and is reviewed in the book by Nield and Bejan (1999). The $1/2$ exponent was predicted for both natural convection and forced convection.

Another common feature of animal hair coats is the porosity, which is high and nearly constant (between 0.95 and 0.99) for all animal sizes (Bejan, 1993). This feature was predicted by minimizing the combined heat loss by conduction and radiation through the hair air coat. This analysis and the most recent design applications of the constructal law are reviewed in Bejan et al. (2004).

Constructal theory predicts not only the $3/4$ exponent for Eqn 6, but also the gradual decrease of this exponent as the body size decreases. The $3/4$ exponent is valid at the limit where the body heat loss is impeded primarily by the convective resistance posed by the blood counterflow of perfectly matched tube pairs (Fig. 2B). As shown in Bejan (2000, 2001), heat-loss paths in general are more complicated. The convective thermal resistance posed by the trees in counterflow (R_1) resides inside the animal. This resistance runs in parallel with a second internal resistance (R_2) associated with the conductive heat leak through the tissue. On the outside of the animal the heat current flows through the convective resistance (R_3) associated with the body surface exposed to the ambient (air, water). The conductive resistance R_2 is proportional to the body length scale $V^{1/3}$ divided by the body surface scale $V^{2/3}$, hence $R_2 \sim V^{-1/3}$. The convective tree resistance R_1 is proportional to $V^{-3/4}$. The ratio $R_2/R_1 \sim V^{5/12}$ shows that R_2 becomes progressively weaker (i.e. the preferred path) as the body size decreases. At that limit the exponent in the power law between heat loss and body size becomes $1/3$. In other words, from constructal theory we should expect a gradual decrease in the power-law exponent as the body size decreases.

The generality of the constructal deduction of the allometric law of metabolism (Eqn 6) is due to the view that the flow structure results from the clash between two objectives: the need to carry certain substances from the core to the periphery of the organism (e.g. nutrients, water, ions, waste products), and the need to avoid the direct leakage of these substances and energy (heat) into the ambient surroundings. All biological flow architectures are results of this clash, from microbes to plants and animals, including warm-blooded and cold-blooded vertebrates. The regulated temperature difference between the

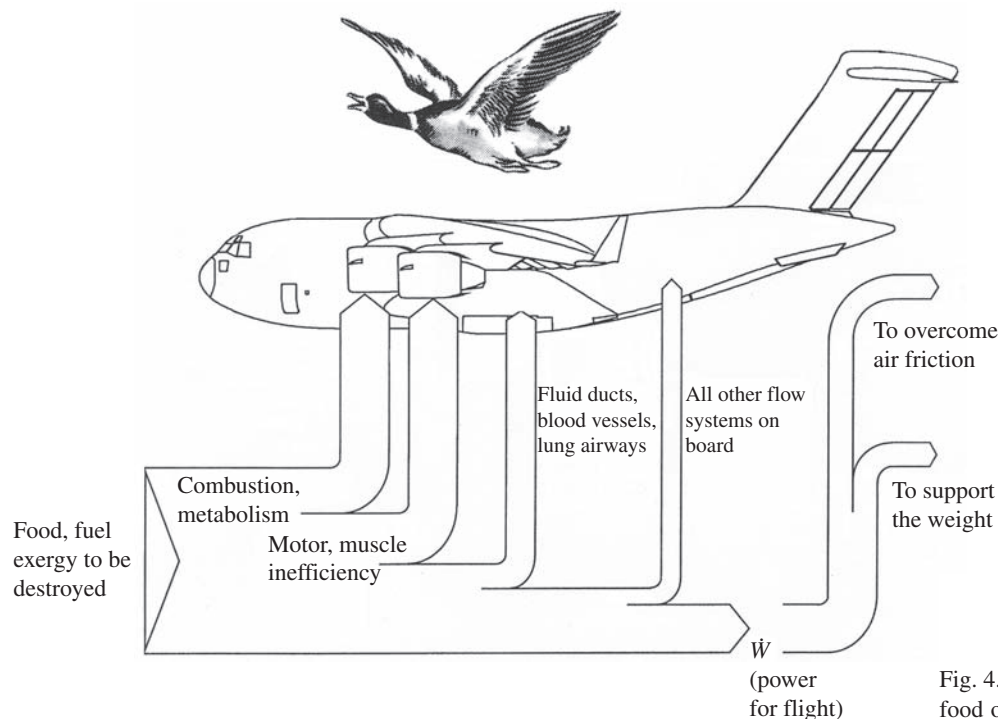


Fig. 4. The distributed destruction of food or fuel exergy during flight.

body core and the ambient surroundings (large, or small) is not the issue. According to constructal theory, the flow system (the animal and its movement) must evolve like any other engine-propelled body on the surface of the earth (e.g. Fig. 5), and this means that the exergy derived from the food must be channeled optimally through the motor (muscles), not dumped straight into the ambient surroundings.

Flight and organ size

The design principle and results reviewed so far are relevant across the board, from biology to engineering. This point is pressed with vigor by the aircraft sketched in Fig. 4: if the word ‘fuel’ is replaced by ‘food’, then the same drawing is valid for a bird, and reveals how the energy liberated by food is destroyed by all the currents that flow around and through the animal. The useful (liberated) energy is known as exergy in thermodynamics (Bejan, 1997a), and as energy consumption in biology. The food or fuel exergy is destroyed completely by currents that overcome resistances.

The flying system becomes ‘more fit’ when the total destruction of exergy is minimized: more body mass flown, to longer distances. This is just like the Gulf Stream: more ocean mass carried with less resistance, i.e. faster and for longer distances. The mechanisms that destroy food exergy (e.g. air friction) cannot be minimized individually and eliminated, because each such mechanism serves the flying body as a

whole. This is illustrated in most general terms (Bejan, 2000) by minimizing the sum (\dot{W}) of the food exergies required by air friction and lifting (supporting) the aircraft:

$$\dot{W} \sim \rho_a D^2 V^3 + \frac{\rho_b^2 g^2 D^4}{\rho_a V}, \quad (11)$$

where D is the body linear dimension, V is cruising speed, g is gravitational acceleration, ρ_a is air density and ρ_b is body density (based on the total body mass scale $M_b \sim \rho_b D^3$). The terms on the right side of Eqn 11 cannot be eliminated. They can be minimized together, thus:

$$\dot{W}_{\min} \sim \frac{\rho_b^{1/3} g^{3/2}}{\rho_a^{1/2}} M_b^{7/6}, \quad (12)$$

when the cruising speed V_{opt} has the scale

$$V_{\text{opt}} \sim \rho_b^{1/3} \left(\frac{g}{\rho_a} \right)^{1/2} M_b^{1/6}. \quad (13)$$

Flying animals and machines are represented by the scales $\rho_b \sim 10^3 \text{ kg m}^{-3}$ and $\rho_a \sim 1 \text{ kg m}^{-3}$, such that Eqn 13 becomes:

$$V_{\text{opt}} \sim 30 M_b^{1/6}, \quad (14)$$

where V_{opt} is in m s^{-1} and M_b is in kg .

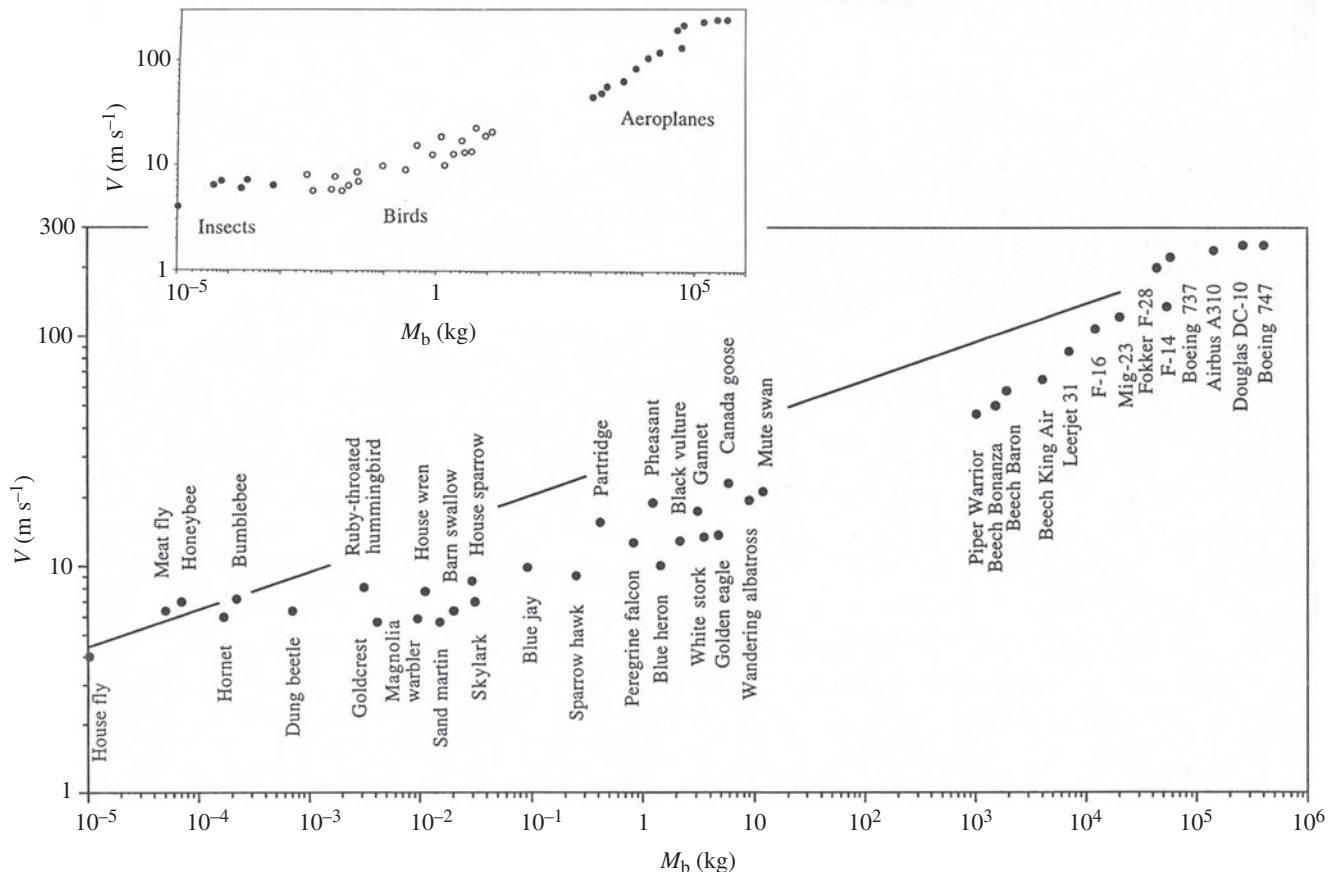


Fig. 5. The flying speeds of insects, birds and aeroplanes, and their theoretical speed (Bejan, 2000).

Fig. 5 shows this theoretical line next to flying speed data taken from extensive compilations (Tennekes, 1996, and references therein). The agreement between the line and the data is remarkable, in view of the simplicity of the body model with one length scale (D). Insects, birds and aeroplanes have multiple length scales, and this may explain why some of the data fall above or below the line.

Agreement over such a wide diversity of sizes and types of flying flow systems shows that the constructal law — the optimal distribution of imperfection — unites the designs of all the flying systems, the animate with the engineered. This is stressed by the additional features of Fig. 5. Small animals (insects, hummingbirds) flap their wings all the time, and their engine propellers (the wings) also provide the lift. In this limit of small mass, the motor and the lift functions are performed by a single structure: the wings. At the other end of the body mass scale, large masses (aircraft) fly with separate motor and lift structures. The lift is provided by the wings proper, and the motor (thrust) by a different set of wings — the blades of the turbofan engine.

Between the ‘fully integrated’ and ‘separate’ motor and lift we find the ‘almost separate’ distribution of motor and lift functions. We see this in the V-shaped flocks of migratory birds. The goose is the motor when it flies at or near the tip of the V; when it is not, the goose surfs on the waves generated by the geese working in front. Pterosaurs are also in-between. Their motor and lift functions were almost separate: they flapped their wings rarely, and glided most of the time under the hot sun (Frey et al., 2003).

More recently, we showed that the minimized food consumption dictates the physical sizes of the various flow components of a complex flying system (Bejan and Lorente, 2002). Consider the total food required for flying over a distance L . This quantity is proportional to the total food exergy that is destroyed, $W \sim \dot{W}_{\min} L / V_{\text{opt}}$ which, according to Eqns 12 and 13, is proportional to body weight and distance:

$$W \sim M_b g L. \quad (15)$$

In the aircraft industry this proportionality is known as the ‘take-off gross weight’ criterion: the fuel penalty associated with placing a new component on board is proportional to the mass of that component. Smaller flow components are attractive. But, flow components function less efficiently when their sizes decrease: they pose greater resistance to flows, they destroy more exergy, and so the flying animal or machine requires more food or fuel to carry such components.

Constructal theory accounts for the existence of characteristic (proportionate) organ sizes, in animals and engineering installations. The fundamental trade-off in body size is illustrated in Fig. 6. The total food exergy required by a flow component is the sum of the food exergy destroyed by the component and the food exergy required by the flying system to carry the component on board. There is an optimal component size such that its impact (penalty) on the total food required by the bird is minimum. This trade-off is fundamental: it rules the optimization of organ sizes in every flow system,

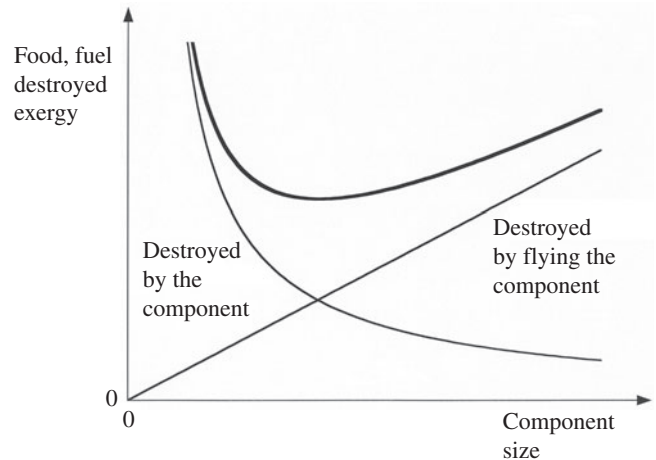


Fig. 6. Minimization of the total food or fuel requirement associated with one component in a complex flow system.

animals and vehicles alike. A simple illustration of this basic phenomenon is the optimization of the diameter D of a round duct (pipe or blood vessel), when its mass flow rate \dot{m} is prescribed (Bejan and Lorente, 2002).

Survival by increasing performance, compactness and territory

In constructal theory, body size, architecture and complexity are results, not assumptions. They are intrinsic parts of the drawing: the optimal configuration to which the flow system tends in time, in accordance with the constructal law. This tendency was recently put on an analytical basis, such that the constructal law becomes a new extension of thermodynamics — the thermodynamics of non-equilibrium (flow) systems with configuration (Bejan and Lorente, 2004). This formulation is condensed in Fig. 7. A flow system (e.g. a tree) has ‘properties’ that distinguish it from a static (non-flow) system. The properties of a flow system are: (1) global external size, e.g. the length scale of the body bathed by the tree flow, L ; (2) global internal size, e.g. the total volume of the ducts, V ; (3) at least one global objective, or performance, e.g. the global flow resistance of the tree, R ; (4) configuration, drawing, architecture; and (5) freedom to morph, i.e. freedom to change the configuration.

The flow systems covered by the constructal law stated at the start of this article populate and move in the V =constant plane shown in Fig. 7. This plane houses a Performance vs Freedom diagram: in time, and if the architecture is free to change, R decreases (i.e. performance increases) at constant L and V . The configuration with the smallest R value represents the equilibrium flow structure. The configurations that preceded it are non-equilibrium flow structures.

At equilibrium the flow configuration achieves the most that its freedom to morph has to offer. Equilibrium does not mean that the flow architecture stops changing. On the contrary, it is here at equilibrium that the flow geometry enjoys most

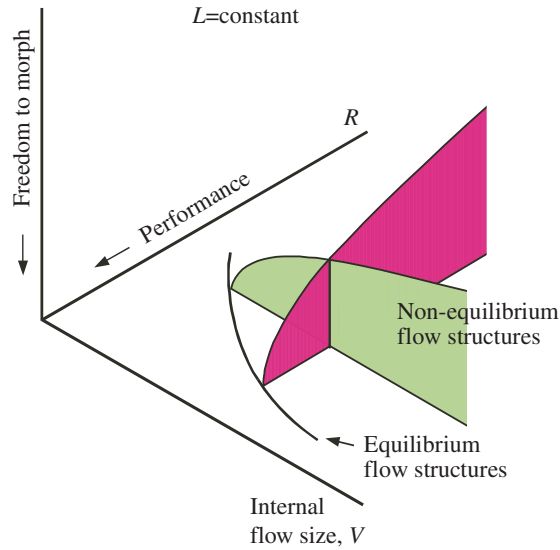


Fig. 7. Performance vs Freedom to change configuration, at fixed global size L .

freedom to change. Equilibrium means that the global performance does not change when changes occur in the flow architecture.

The evolution of configurations in the constant- V cut (also at constant L , Fig. 7) represents survival through increasing performance — survival of the fittest. This is the physics principle that now underpins Darwin’s argument, the law that rules not only the animate flow systems but also the natural inanimate flow systems and all the man and machine species. The constructal law defines the meaning of ‘the survivor’, or of the equivalent concept of ‘the more fit’.

In the bottom plane of Fig. 7, the locus of equilibrium structures is a curve with negative slope, $(\partial R/\partial V) < 0$, because of flow physics: the resistance decreases when the size of the internal space inhabited by the flow increases. This slope means that the non-equilibrium flow structures occupy the hypersurface suggested by the three-dimensional surface sketched in Fig. 7. The time evolution of non-equilibrium flow structures toward the bottom edge of the surface (the equilibrium structures) is the action of the constructal law.

The same time arrow can be described alternatively with reference to the constant- R cut through the three-dimensional space of Fig. 7. Flow architectures with the same global performance (R) and global size (L) evolve toward compactness — smaller volumes dedicated to internal ducts, i.e. larger volumes reserved for the working ‘tissue’ (the interstices). This is survival based on the maximization of the use of the available space. Survival *via* increasing compactness is equivalent to survival *via* increasing performance: both statements are the constructal law.

A third equivalent statement of the constructal law becomes evident if we recast the constant- L design world of Fig. 7 in the constant- V design space of Fig. 8. In this new figure, the constant- L cut is the same Performance vs Freedom diagram

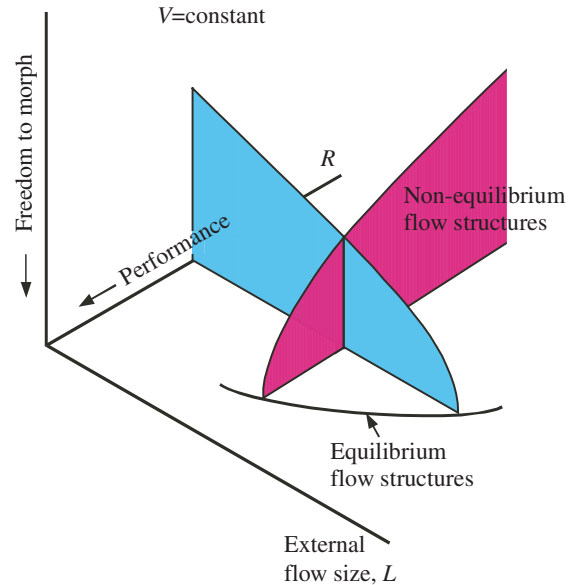


Fig. 8. Performance vs Freedom to change configuration, at fixed internal flow size V .

as in Fig. 7, and the constructal law means survival by increasing performance. The new aspect of Fig. 8 is the shape and orientation of the hypersurface of non-equilibrium flow structures: the slope of the curve in the bottom plane $(\partial R/\partial L)_V$ is positive because of flow physics, i.e. because the flow resistance increases when the distance traveled by the stream increases.

The world of possible designs (the hypersurface) can be viewed in the constant- R cut made in Fig. 8, to see that flow structures of a certain performance level (R) and internal flow volume (V) morph into new flow structures that cover progressively larger territories. There is a limit to the spreading of a flow structure, and it is set by global properties such as R and V . River deltas in the desert, animal species on the plain, and the Roman empire spread to their limits. Such is the constructal law of survival by spreading, by increasing territory for flow and movement.

Overview: theory vs modeling

This article began with a broad view of how the constructal law accounts for the generation of configuration everywhere, from flow architectures to mechanical structures, and from animate systems (biology), to inanimate systems (physics) and man + machine species (engineering, economics, business). The constructal literature listed in the References covers all these domains: this literature is recommended to the readers because, although known in engineering, it is largely unknown in biology.

The applications of constructal theory are not limited to the examples given in this article. In fact, the connections between constructal theory and the large volume of observations available in established fields such as biology are yet to be

made. They deserve to be explored to depths much greater than in engineering books (Bejan, 1997a, 2000). In particular, because this special issue focuses on allometric laws and body size, it is important to see clearly the position of constructal theory relative to the allometric model of West et al. (1997).

Constructal theory (Bejan, 1996, 1997a–c) predates the model of West et al. (1997), and covers a physics that is much greater than the allometric behavior correlated by the model. As stated in the title of their paper, the work of West et al. is the modeling of certain biological systems, which they observe and then describe analytically. Modeling is empiricism, not theory.

The model of West et al. is based on at least three *ad hoc* assumptions: (i) the existence of a ‘space-filling fractal-like branching pattern’ (read: tree), (ii) the final branch of the network is a size-invariant unit, and (iii) the energy required to distribute resources is minimized.

These three features were already present in 1996 constructal theory, not as convenient assumptions to polish a model and make it work, but as invocations of a single principle: the constructal law. Specifically, (iii) is covered by the constructal law, (i) is the tree-flow architecture that in constructal theory is deduced from the constructal law, and (ii) is the smallest-element scale that is fixed in all the constructal tree architectures. To repeat, in constructal theory the tree-shaped flow is a discovery, not an observation, and not an assumption.

Because features (i–iii) are shared by constructal theory and by the model of West et al., every allometric law that West et al. connect to their model is an affirmation of the validity of constructal theory. Every success of constructal theory in domains well beyond the reach of their model (e.g. river basins, flight, dendritic solidification, global circulation, mud cracks; see Bejan, 2000) is an indication that animal design is an integral part of a general theoretical framework — a new thermodynamics — that unites biology with physics and engineering.

Biology and ‘natural’ selection have just been made a part of physics. There are two time arrows in this new thermodynamics. The old is the time arrow of the second law, i.e. the statement of irreversibility: everything flows from high to low. The new is the time arrow of the constructal law, or ‘how’ everything flows: configurations morph toward easier flowing architectures, toward animal designs that are more fit, toward geophysical currents that flow along better paths, and toward man + machine species that are more efficient. All these macroscopic constructs mix the earth’s crust better and better, that is much more effectively than in the absence of flow architecture.

Constructal theory strikes a balance between determinism and chance. In a constructal tree, for example, the position of the branches can be predicted (e.g. Fig. 1), but nobody knows exactly how the individual or the molecule will move across the interstices. Likewise, it is not known exactly how (i.e. in what vertical plane of Figs 7 and 8) a non-equilibrium flow structure migrates toward equilibrium. Chance and additional constraints will definitely play a role. There is no ambiguity, however, about the direction of the migration, and about the

top performance level, which is achieved at equilibrium, where imperfections are distributed optimally.

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