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Sprint running: a new energetic approach

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Summary

The speed of the initial 30 m of an all-out run from a stationary start on a flat track was determined for 12 medium level male sprinters by means of a radar device. The peak speed of 9.46±0.19 m s^{-1} (mean ± S.D.) was attained after about 5 s, the highest forward acceleration (a_f), attained immediately after the start, amounting to 6.42±0.61 m s^{-2}. During acceleration, the runner's body (assumed to coincide with the segment joining the centre of mass and the point of contact foot terrain) must lean forward, as compared to constant speed running, by an angle α=arctang/a_f (g=acceleration of gravity). The complement (90–α) is the angle, with respect to the horizontal, by which the terrain should be tilted upwards to bring the runner's body to a position identical to that of constant speed running. Therefore, accelerated running is similar to running at constant speed up an 'equivalent slope' ES=tan(90–α). Maximum ES was 0.643±0.059. Knowledge of ES allowed us to estimate the energy cost of sprint running (C_{sr}, J kg^{-1} m^{-1}) from literature data on the energy cost measured during uphill running at constant speed. Peak C_{sr} was 43.8±10.4 J kg^{-1} m^{-1}; its average over the acceleration phase (30 m) was 10.7±0.59 J kg^{-1} m^{-1}, as compared with 3.8 for running at constant speed on flat terrain. The corresponding metabolic powers (in W kg^{-1}) amounted to 91.9±20.5 (peak) and 61.0±4.7 (mean).

Key words: sprint, running, muscle energetics, human.

Introduction

Since the second half of the 19th century, the energetics and biomechanics of running at constant speed have been the object of many studies, directed towards elucidating the basic mechanisms of this most natural form of locomotion; but the results of these studies have also had direct practical applications, e.g. for the assessment of the overall metabolic energy expenditure, or for the prediction of best performances (e.g. see Alvarez-Ramirez, 2002; Lacour et al., 1990; Margaria, 1938; Margaria et al., 1963; Péronnet and Thibault, 1989; di Prampero et al., 1993; Ward-Smith, 1985; Ward-Smith and Mobey, 1995; Williams and Cavanagh, 1987).

In contrast to constant speed running, the number of studies devoted to sprint running is rather scant. This is not surprising, since the very object at stake precludes reaching a steady state, thus rendering any type of energetic analysis rather problematic. Indeed, the only published works on this matter deal with either some mechanical aspects of sprint running (Cavagna et al., 1971; Fenn, 1930a,b; Kersting, 1998; Mero et al., 1992; Murase et al., 1976; Plamondon and Roy, 1984), or with some indirect approaches to its energetics (Arsac, 2002; Arsac and Locatelli, 2002; van Ingen Schenau et al., 1991, 1994; di Prampero et al., 1993; Summers, 1997; Ward-Smith and Radford, 2000). The indirect estimates of the metabolic cost of acceleration reported in the above-mentioned papers are based on several assumptions that are not always convincing. In the present study we therefore propose a novel approach to estimate the energy cost of sprint running, based on the equivalence of an accelerating frame of reference (centred on the runner) with the Earth's gravitational field. Specifically, in the present study, sprint running on flat terrain will be viewed as the analogue of uphill running at constant speed, the uphill slope being dictated by the forward acceleration (di Prampero et al., 2002). Thus, if the forward acceleration is measured, and since the energy cost of uphill running is fairly well known (e.g. see Margaria, 1938; Margaria et al., 1963; Minetti et al., 1994, 2002), it is a rather straightforward matter to translate the forward acceleration of sprint running into the corresponding up-slope, and thence into the corresponding energy cost. Knowledge of this last and of the instantaneous forward speed will then allow us to calculate the corresponding metabolic power, which is presumably among the highest values attainable for any given subject.

Theory

In the initial phase of sprint running, the overall acceleration acting on the runner's body (**g′**) is the vectorial sum of the

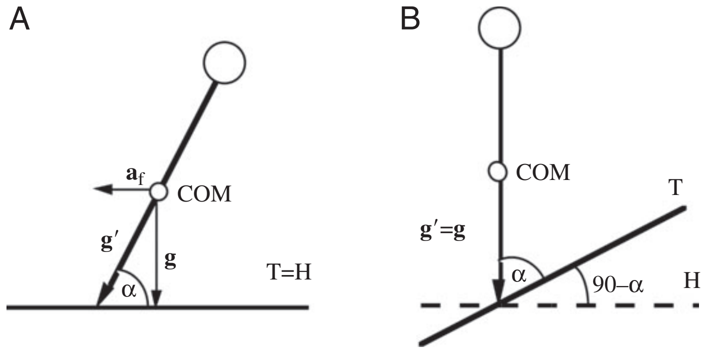

Fig. 1. Simplified view of the forces acting on a runner. The subject is accelerating forward while running on flat terrain (A) or running uphill at constant speed (B). The subject's body mass is assumed to be located at the centre of mass (COM); $\mathbf{a}_f$=forward acceleration; $\mathbf{g}$=acceleration of gravity; $\mathbf{g}'=(\mathbf{a}_f^2+\mathbf{g}^2)^{0.5}$ is the acceleration resulting from the vectorial sum of $\mathbf{a}_f$ plus $\mathbf{g}$; T=terrain; H=horizontal; α (=arctan $\mathbf{g}/\mathbf{a}_f$) is the angle between runner's body and T; the angle between T and H is $\alpha'=90-\alpha$. (Modified from di Prampero et al., 2002.)

forward acceleration ($\mathbf{a}_f$) and the Earth's acceleration of gravity ($\mathbf{g}$), both assumed to be applied to the subject's centre of mass (COM; Fig. 1A):

$$\mathbf{g}' = (\mathbf{a}_f^2 + \mathbf{g}^2)^{0.5} . \quad (1)$$

To maintain equilibrium, the angle α between $\mathbf{g}'$ (which is applied along a line joining the point of contact foot–terrain with the runner's body COM) and the terrain must be given by:

$$\alpha = \arctan \mathbf{g} / \mathbf{a}_f . \quad (2)$$

This state of affairs is analogous to that applying if the subject were running uphill at constant speed, in which case the overall average acceleration ($\mathbf{g}'$) is assumed to be applied vertically (Fig. 1B). Indeed, if $\mathbf{g}'$ is tilted upwards, so as to render it vertical, to maintain constant the angle of $\mathbf{g}'$ with the terrain (α), the latter must also be tilted upwards, with respect to the horizontal, by the same amount. Inspection of Fig. 1 makes it immediately apparent that the angle between the horizontal and the terrain (α'), due to the forward acceleration yielding the angle α between $\mathbf{g}'$ and the terrain, is given by:

$$\alpha' = 90 - \alpha = 90 - \arctan \mathbf{g} / \mathbf{a}_f . \quad (3)$$

The slope equivalent to the angle α' (equivalent slope, ES) is therefore given by the tangent of the angle α' itself:

$$\mathrm{ES} = \tan (90 - \arctan \mathbf{g} / \mathbf{a}_f) . \quad (4)$$

In addition, during sprint running, the average force exerted by active muscles during the stride cycle ($\mathbf{F}'$=equivalent body weight) is given by:

$$\mathbf{F}' = M_b\mathbf{g}' , \quad (5)$$

where M_b is the runner's body mass. When running at constant speed, the average force ($\mathbf{F}$) corresponds to the subject's body weight:

$$\mathbf{F} = M_b\mathbf{g} . \quad (6)$$

The ratio of Eq. 5 to Eq. 6

$$\mathbf{F}' / \mathbf{F} = \mathbf{g}' / \mathbf{g} \quad (7)$$

shows that, during sprint running, the equivalent body weight ($\mathbf{F}'$=the average force generated by the active muscles) is equal to that required to transport, at constant speed on the Earth, the same mass (M_b) multiplied by the ratio $\mathbf{g}'/\mathbf{g}$. This ratio will here be called 'equivalent normalised body mass' (EM). Thus, from Eq. 1:

$$\mathrm{EM} = \mathbf{g}' / \mathbf{g} = (\mathbf{a}_f^2 / \mathbf{g}^2 + 1)^{0.5} . \quad (8)$$

Summarising, sprint running can be considered equivalent to constant speed running on the Earth, up an equivalent slope ES, while carrying an additional mass $\Delta M = M_b(\mathbf{g}'/\mathbf{g}-1)$, so that the overall equivalent mass EM becomes $\mathrm{EM}=\Delta M+M_b$.

Both ES and EM are dictated by the forward acceleration (Eq. 4, 8); therefore they can be easily calculated once $\mathbf{a}_f$ is known. The values of ES and EM so obtained can then be used to infer the corresponding energy cost of sprint running, provided that the energy cost of uphill running at constant speed per unit body mass is also known.

It should be pointed out that the above analogy is based on the following three simplifying assumptions, which will be discussed in the appropriate sections. (i) Fig. 1 is an idealised scheme wherein the overall mass of the runner is assumed to be located at the centre of mass. In addition, (ii) Fig. 1 refers to the whole period during which one foot is on the ground, as such it denotes the integrated average applying to the whole step (half stride). (iii) The calculated ES and EM values are those in excess of the values applying during constant speed running, in which case the subject's body is not vertical, but leans slightly forward (Margaria, 1975).

Aims

The aim of the present study was that to estimate the energy cost and metabolic power of the first 30 m of an all-out run from a stationary start, from the measured forward speed and acceleration.

Methods and calculations

The experiments were performed on an outdoor tartan track of 100 m length, at an average barometric pressure and temperature of about 740 mmHg and 21°C, using 12 medium-level male sprinters whose physical characteristics are reported in Table 1. The subjects were informed on the aims of the study and gave their written consent to participate.

The instantaneous speed of the initial 30 m of an all-out run from regular starting blocks was continuously determined by

Table 1. *Physical characteristics of subjects and best performance times over 100 m during the coeval agonistic season*

Subject	Age (years)	Body mass (kg)	Stature (m)	Best performance (s)
1	19	74.0	1.78	11.52
2	24	82.0	1.80	11.13
3	18	66.0	1.75	10.90
4	26	84.0	1.92	10.96
5	19	82.0	1.83	12.09
6	21	70.0	1.79	11.45
7	24	68.0	1.72	11.04
8	21	66.0	1.71	11.06
9	21	72.0	1.80	11.02
10	21	84.0	1.87	11.28
11	18	72.0	1.85	11.66
12	18	70.0	1.78	11.50
Mean	21.0	74.2	1.80	11.30
S.D.	2.7	7.0	0.06	0.35

means of a radar Stalker ATS System™ (Radar Sales, Minneapolis, MN, US) at a sampling frequency of 35 Hz. Raw speed data were filtered (by a fourth order, zero lag, Butterworth filter) using the ATS System™ acquisition software. The radar device was placed on a tripod 10 m behind the start line at a height of 1 m, corresponding approximately to the height of the subject's center of mass. To check the reliability of the radar device, the 12 subjects performed an entire 100 m run. The times obtained on each 10 m section (t_{radar}) were compared to those obtained over the same sections by means of a photocell system (t_{cells}). The two sets of data were essentially identical:

$$t_{radar} = 1.01t_{cells} - 0.06;\ r^2 = 0.99;\ N=120;\ P<0.01\ , \quad (9)$$

thus confirming a previous validation carried out by Chelly and Denis (2001) on moving objects.

The speed–time curves were then fitted by an exponential function (Chelly and Denis, 2001; Henry, 1954; Volkov and Lapin, 1979):

$$\mathbf{s}(t) = \mathbf{s}_{max} * (1-e^{-t/\tau})\ , \quad (10)$$

where **s** is the modelled running speed, $\mathbf{s}_{max}$ the maximal velocity reached during the sprint, and τ the time constant.

Typical tracings of the measured or modelled speeds so obtained are reported in Fig. 2 as a function of time. Since the exponential model described the actual running speeds accurately (see Discussion and Fig. 3), the instantaneous forward acceleration was then calculated from the first derivative of Eq. 10:

$$\mathbf{a}_f(t) = d\mathbf{s} / dt = [\mathbf{s}_{max} - \mathbf{s}_{max} * (1-e^{-t/\tau})] / \tau\ . \quad (11)$$

This is plotted in Fig. 4 as a function of the distance (*d*, m) of the run, as obtained from the time integral of Eq. 10:

$$d(t) = \mathbf{s}_{max} * t - [\mathbf{s}_{max} * (1-e^{-t/\tau})] * \tau\ . \quad (12)$$

The individual values of speed and acceleration were calculated for each subject over one run. The values so

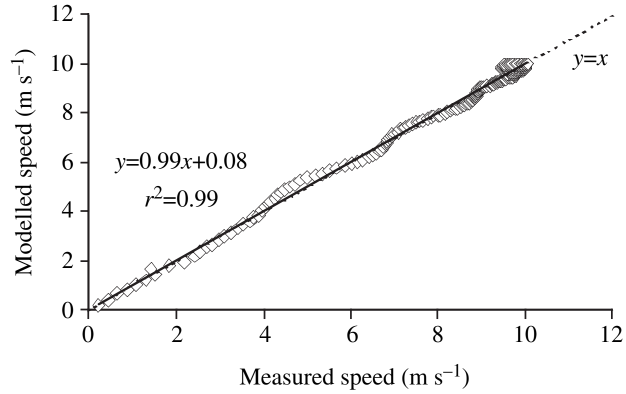

Fig. 3. Running velocity as calculated by the exponential model, as a function of the actual running speed for Subject 7. The linear relationship is reported in the figure (*N*=234); identity line is also shown.

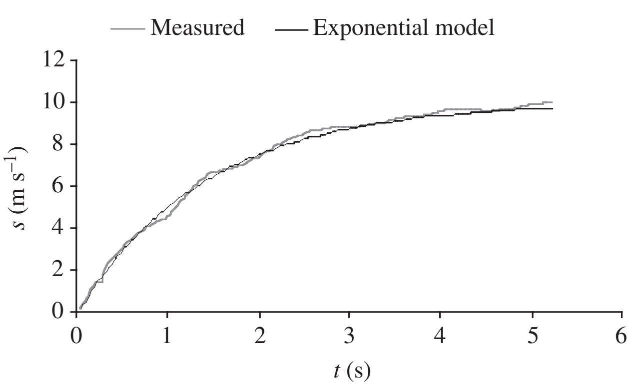

Fig. 2. Actual (gray, thick line) and modelled (black, thin line) forward speed **s** (m s^{-1}) as a function of time *t* (s) at the onset of a typical 100 m run for subject 7. Actual speed was accurately described by: $\mathbf{s}(t)$=10.0*(1–e$^{-t/1.42}$). The maximal speed ($\mathbf{s}_{max}$) was 10.0 m s^{-1}.

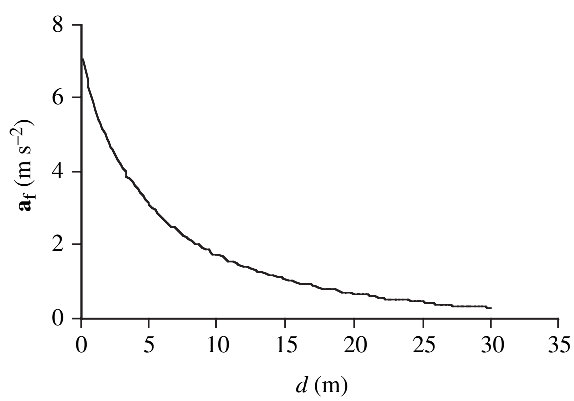

Fig. 4. The instantaneous forward acceleration $\mathbf{a}_f$ (m s^{-2}), obtained as described in the text, is plotted as a function of the distance *d* (m) for subject 7.

obtained were then pooled and the means calculated. Values are reported as means ± 1 standard deviation (S.D.), where N=12.

The individual values of ES (Eq. 4) and EM (Eq. 8) were also obtained for all subjects from the forward acceleration. This allowed us to calculate the energy cost of sprint running with the aid of the data of literature. Indeed, as reported by Minetti et al. (2002) for slopes from –0.45 to +0.45, the energy cost of uphill running per unit of distance along the running path C (J kg^{-1} m^{-1}), is described by:

$$C = 155.4x^5 - 30.4x^4 - 43.3x^3 + 46.3x^2 + 19.5x + 3.6\ , \quad (13)$$

where x is the incline of the terrain, as given by the tangent of the angle α' with the horizontal (see Eq. 3 and Fig. 1B). Thus, the estimated energy cost of sprint running (C_{sr}) can be calculated replacing x in the above equation with the calculated values of ES (Eq. 4) and multiplying the sum of the indicated terms by EM (Eq. 8):

$$C_{sr} = (155.4\mathrm{ES}^5 - 30.4\mathrm{ES}^4 - 43.3\mathrm{ES}^3 + 46.3\mathrm{ES}^2 + 19.5\mathrm{ES} + 3.6)\mathrm{EM}\ . \quad (14)$$

It is also immediately apparent that, when ES=0 and EM=1, C_{sr} reduces to that applying at constant speed running on flat terrain, which amounted to about 3.6 J kg^{-1} min^{-1} (Minetti et al., 2002), a value close to that reported by others (e.g. see Margaria et al., 1963; di Prampero et al., 1986, 1993).

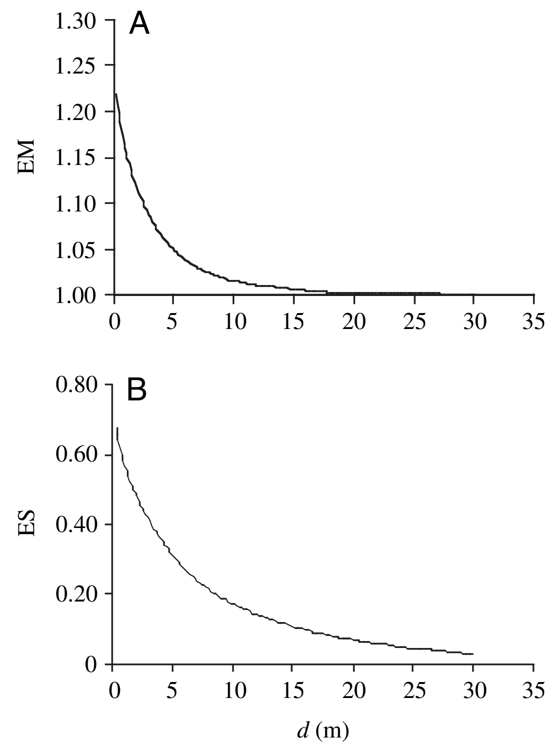

Fig. 5. Equivalent body mass (EM; A) and equivalent slope (ES; B), as a function of the distance d (m) for subject 7.

Results

The speed increased to attain a peak of 9.46±0.19 m s^{-1} about 5 s from the start. The highest forward acceleration was observed immediately after the start (0.2 s): it amounted to 6.42±0.61 m s^{-2}. The corresponding peak ES and EM values amounted to 0.64±0.06 and to 1.20±0.03 (Table 2). The behavior of ES and EM, throughout the entire acceleration phase for a typical subject, as calculated from $\mathbf{a}_f$ (see Fig. 4) on the bases of Eq. 4 and 8, is reported in Fig. 5, which shows that, after about 30 m, ES tended to zero and EM to one, which correspond to constant speed running.

The energy cost of sprint running (C_{sr}), as obtained from Eq. 13 on the basis of the above calculated ES and EM, is reported in Fig. 6 for a typical subject. This figure shows that the instantaneous C_{sr} attains a peak of about 50 J kg^{-1} m^{-1} immediately after the start; thereafter it declines progressively to attain, after about 30 m, the value for constant speed running on flat terrain (i.e. about 3.8 J kg^{-1} m^{-1}). This figure shows also that ES is responsible for the greater increase of C_{sr} whereas EM plays only a marginal role. Finally, Fig. 6 also shows that the average C_{sr} over the first 30 m of sprint running in this subject is about 11.4 J kg^{-1} m^{-1}, i.e. about three times larger than that of constant speed running on flat terrain.

The product of C_{sr} and the speed yields the instantaneous metabolic power output above resting; it is reported as a

Table 2. *Grand averages of peak values of speed (**s**), forward acceleration (**a**$_f$), equivalent slope (ES) and equivalent body mass (EM)*

	s (m s^{-1})	**a**$_f$ (m s^{-2})	ES	EM
Mean	9.46	6.42	0.64	1.20
S.D.	0.19	0.61	0.06	0.03
CV	0.020	0.095	0.091	0.025

S.D., standard deviations; CV, coefficient of variation. N=12 throughout.

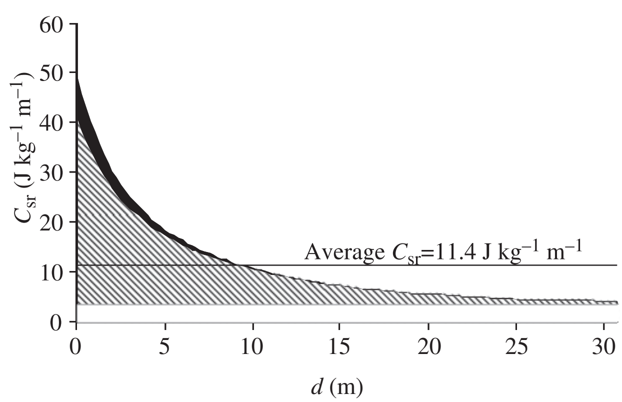

Fig. 6. Energy cost of sprint running C_{sr} (J kg^{-1} m^{-1}), as calculated by means of Eq. 14, as a function of the distance d (m) for subject 7. Energy cost of constant speed running is indicated by the lower horizontal thin line. Black and hatched distances between appropriate lines indicate effects of EM and ES, respectively. Upper horizontal thin line indicates average C_{sr} throughout the indicated distance.

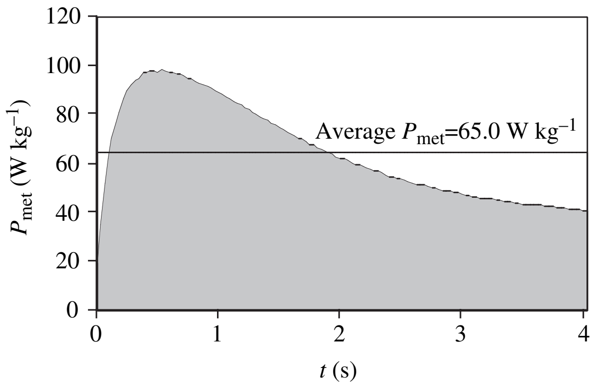

Fig. 7. Metabolic power P_{met} (W kg^{-1}), as calculated from the product of C_{sr} (see Fig. 6) and the speed, as a function of time t (s) for subject 7. Average power over 4 s is indicated by horizontal thin line.

function of time for the same subject in Fig. 7, which shows that the peak power output, of about 100 W kg^{-1}, is attained after about 0.5 s and that the average power over the first 4 s is on the order of 65 W kg^{-1}.

Discussion

Critique of methods

The instantaneous values of forward acceleration were obtained from the first derivative of exponential equations describing the time course of the speed. Linear regressions between measured and modelled speed values (Fig. 3) were close to the identity line for all 12 subjects (r^2>0.98; P<0.01), showing the high accuracy of this kind of speed modelling during sprint running (Chelly and Denis, 2001; Henry, 1954; Volkov and Lapin, 1979). Even so, it should be noted that: (i) at the start of the run the centre of mass is behind the start line and (ii) whereas the centre of mass rises at the very onset of the run, the radar device does not; as a consequence, (iii) the initial speed data are slightly biased. However, after a couple of steps this effect becomes negligible, as such it will not be considered further. Finally, it should also be pointed out that filtering the raw speed data, while retaining the general characteristics of the speed *vs* time curve (Fig. 3), leads to substantial smoothing of the speed swings that occur at each step and are a fundamental characteristic of locomotion on legs.

The number of subjects of this study (12) may appear small. However the coefficients of variation of peak speeds and peak accelerations for this population (0.02 and 0.095) were rather limited, and the subjects were homogeneous in terms of performance (Tables 1, 2). Finally, the present approach is directed at obtaining a general description of sprint running, rather than at providing accurate statistical descriptions of specific groups of athletes.

The main assumptions on which the calculations reported in the preceding sections were based are reported and discussed below.

(1) The overall mass of the runner is assumed to be located at the centre of mass of the body. As such, any possible effects of the motion of the limbs, with respect to the centre of mass, on the energetics of running were neglected. This is tantamount to assuming that the energy expenditure associated with internal work is the same during uphill running as during sprint running at an equal ES. This is probably not entirely correct, since the frequency of motion is larger during sprint than during uphill running. If this is so, the values obtained in this study can be taken to represent a minimal value of the energy cost, or metabolic power, of sprint running.

(2) The average force applied by the active muscles during the period in which one foot is on the ground is assumed to be described as in Fig. 1B, thus neglecting any components acting in the frontal plane. In addition, the assumption is also made that the landing phase (in terms of forces and joint angles) is the same during uphill as during sprint running at similar ES, a fact that may not be necessarily true, and that may warrant *ad hoc* biomechanical studies.

(3) The calculated ES and EM values are assumed to be in excess of those applying during constant speed running, in which case the subject's body is not vertical, but leans slightly forward (Margaria, 1975) and the average force required to transport the runner's body mass is equal to that prevailing under the Earth's gravitational field. Indeed, the main aim of this study was to estimate the energy cost and metabolic power of sprint running, and since our reference was the energy cost of constant speed running per unit body mass, the above simplifying assumptions should not introduce any substantial error in our calculations.

(4) The energy cost of running uphill at constant speed, as measured at steady state up to inclines of +0.45, was taken to represent also the energy cost of sprint running at an equal ES. Note that the energy cost of running per unit of distance, for any given slope, is independent of the speed (e.g. see Margaria et al., 1963; di Prampero et al., 1986; 1993). Thus the transfer from uphill to sprint running can be made regardless of the speed. Even so, the highest values of ES attained by our subjects (about 0.70) were greater than the highest slopes for which the energy cost of uphill running was actually measured (0.45). Thus the validity of our values for slopes greater than 0.45 is based on the additional assumption that, also above this incline, the relationship between C_{sr} and ES is described by Eq. 14. Graphical extrapolation of the Minetti et al. (2002) equation does seem to support our interpretation of their data; however, stretching their applicability as we did in the present study may seem somewhat risky. We would like to point out, however, that the above word of caution applies only for the peak C_{sr} and metabolic power values, i.e. to the initial 3 m (Fig. 5), which represent about 1/10 of the distance considered in this study. Thus, the majority of our analysis belongs to a more conservative range of values.

(5) Minetti et al. (2002) determined the energy cost of uphill running from direct oxygen uptake measurements during aerobic steady state exercise. In contrast, the energy sources of sprint running are largely anaerobic. It follows that the values of C_{sr} and metabolic power (P_{met}), as calculated in this study,

should be considered with caution. Indeed, they are an estimate of the amount of energy (e.g. ATP units) required during the run, expressed in O_2 equivalents. The overall amount of O_2 consumed, including the so-called 'O_2 debt payment' for replenishing the anaerobic stores after the run, may well be different, a fact that applies to any estimate of energy requirement during 'supramaximal exercise'. Finally, the calculated values of C_{sr} and P_{met} represent indirect estimates rather than 'true' measured values. However, the actual amount of energy spent during sprint running cannot be easily determined with present day technology, thus rendering any direct validation of our approach rather problematic. However, in theory at least, computerised image analysis of subjects running over series of force platforms could be coupled with the assessment of the overall heat output by means of thermographic methods. Were this indeed feasible, one could obtain a complete energetic description of sprint running to be compared with the present indirect approach.

Metabolic power of sprint running

The peak metabolic power values reported in Table 3 are about four times larger than the maximal oxygen consumption (V_{O2max}) of elite sprinters which can be expected to be on the order of 25 W kg^{-1} (70 ml O_2 kg^{-1} min^{-1} above resting). This is consistent with the value estimated by Arsac and Locatelli (2002) for sprint elite runners, which amounted to about 100 W kg^{-1}, and with previous findings showing that, on the average, the maximal anaerobic power developed while running at top speed up a normal flight of stairs is about four times larger than V_{O2max} (Margaria et al., 1966).

The same set of calculations was also performed on one athlete (C. Lewis, winner of the 100 m gold medal in the 1988 Olympic games in Seoul with the time of 9.92 s) from speed data reported by Brüggemann and Glad (1990). The corresponding peak values of ES and EM amounted to 0.80 and 1.3, whereas the peak C_{sr} and metabolic power attained 55 J kg^{-1} m^{-1} and 145 W kg^{-1}. The overall amount of metabolic energy spent over 100 m by C. Lewis was also calculated by this same approach. It amounted to 650 J kg^{-1}, very close to that estimated for world record performances by Arsac (2002) and Arsac and Locatelli (2002). However, these same authors, on the basis of a theoretical model originally developed by van Ingen Schenau (1991), calculated a peak metabolic power of 90 W kg^{-1} for male world records, to be compared with the 145 W kg^{-1} estimated in this study for C. Lewis. The model proposed by van Ingen Schenau is based on several assumptions, among which overall running efficiency plays a major role. Indeed, the power values obtained by Arsac and Locatelli (2002) were calculated on the bases of an efficiency (η) increasing with the speed, as described by η_t=0.25+0.25. $\mathbf{v}_t/\mathbf{v}_{max}$ where η_t and $\mathbf{v}_t$ are efficiency and speed at time t, respectively, and $\mathbf{v}_{max}$ is the maximal speed. However, Arsac and Locatelli point out that, if a constant efficiency of 0.228 is assumed, then the estimated peak metabolic power reaches 135 W kg^{-1}, not far from that obtained above for C. Lewis. Thus, in view of the widely different approaches, we think it is the similarity between the two sets of estimated data that should be emphasized, rather than their difference.

Table 3. *Peak and mean energy cost of sprint running and metabolic power for the 12 subjects*

Mean		Peak	
C_{sr}	P_{met}	C_{sr}	P_{met}
(J kg^{-1} m^{-1})	(W kg^{-1})	(J kg^{-1} m^{-1})	(W kg^{-1})
10.7+0.59	61.0±4.66	43.8±10.4	91.9±20.5

C_{sr}, energy cost of sprint running; P_{met}, metabolic power.

Values are means ± S.D. Mean C_{sr} was calculated over 30 m and mean P_{met} over 4 s.

Energy balance of sprint running

It is now tempting to break down the overall energy expenditure of 650 J kg^{-1} needed by C. Lewis to cover 100 m in 9.92 s, into its aerobic and anaerobic components. To this end we will assume that the maximal O_2 consumption (V_{O2max}) of an élite athlete of the calibre of Lewis amounts to 25 W kg^{-1} (71.1 ml O_2 kg^{-1} min^{-1}) above resting. We will also assume that the overall energy expenditure (E_{tot}) is described by:

$$E_{tot} = A_{ns} + V_{O2max}t_e - V_{O2max}(1 - e^{-te/\tau})\tau\ , \qquad (15)$$

where t_e is the performance time, A_{ns} is the amount of energy derived from anaerobic stores utilisation and τ is the time constant of the V_{O2} response at the muscle level (Wilkie, 1980; di Prampero, 2003).

The last term of this equation is the O_2 debt incurred up to the time t_e, because V_{O2max} is not reached instantaneously at work onset, but with a time constant τ; therefore, the overall amount of energy that can be obtained from aerobic energy sources is smaller than the product $V_{O2max}t_e$, by the quantity represented by the third term of the equation. In the literature, the values assigned to τ range from 10 s (Wilkie, 1980; di Prampero et al., 1993) to 23 s (Cautero et al., 2002). So, since in case of C. Lewis, E_{tot}=650 J kg^{-1} and V_{O2max}=25 W kg^{-1}; *Ans* (calculated by Eq. 15) ranged from about 560 J kg^{-1} (for τ=10 s) to about 600 J kg^{-1} (for τ=23 s). Thus, for an élite athlete to cover 100 m at world record speed the anaerobic energy stores must provide an amount of energy on the order of 580 J kg^{-1}. Unfortunately we cannot partition this amount of energy into that produced from lactate accumulation and that derived from splitting phosphocreatine (PCr). However, we can set an upper limit to the maximal amount of energy that can be obtained from A_{ns} as follows. Let us assume that the maximal blood lactate concentration in an élite athlete can attain 20 mmol l^{-1}. Thus, since the accumulation of 1 mmol l^{-1} lactate in blood is energetically equivalent to the consumption of 3 ml O_2 kg^{-1} (see di Prampero and Ferretti, 1999), the maximal amount of energy that can obtained from lactate is about:

$$20 \times 3 \times 20.9 \approx 1250 \text{ J kg}^{-1}\ , \qquad (16)$$

(where 20.9 J ml^{-1} is the energetic equivalent of O_2). The maximal amount of PCr that can be split from rest to exhaustion in an all-out effort can be estimated to be about 22 mmol kg^{-1} of fresh muscle (see Francescato et al., 2003). We can assume that the muscle mass involved in the all-out effort in question, for an élite sprinter, is about 25% of his body mass (e.g. about 25 kg of muscle). If this is so, and since to spare 1 mmol O_2 the amount of PCr that needs to be split is about 6 mmol, which corresponds to a P/O_2 ratio of 6.0, the amount of energy yielded per kg body mass by complete splitting of PCr in the maximally active muscles can be calculated as:

$$0.25 \times 22 \times 1/6 \times 22.4 \bullet 20.9 \approx 430 \text{ J kg}^{-1}, \quad (17)$$

where 22.4 is the volume (ml, STPD) of 1 mmol O_2. Thus the maximal amount of energy that can be obtained at exhaustion from the complete utilisation of anaerobic stores amounts to:

$$1250 + 430 = 1680 \text{ J kg}^{-1}. \quad (18)$$

It can be concluded that the amount of energy derived from A_{ns} during a 100 m dash in a top athlete is about 1/3 of the total, which is consistent with the fact that longer events (200 m or 400 m) are covered at essentially the same, largely anaerobic, speed.

Of wind and down-slopes

In the preceding paragraphs, the effects of the air resistance on the energy cost of sprint running were neglected; they will now be briefly discussed. The energy spent against the air resistance per unit of distance (C_{aer}) increases with the square of the air velocity (**v**): $C_{aer}=k'v^2$, where the proportionality constant (k') amounts to about 0.40 J s^2 m^{-3} per m^2 of body surface area (Pugh, 1971; di Prampero et al., 1986, 1993). This allowed us to calculate that C_{aer} attained about 0.86 J kg^{-1} m^{-1} at the highest speeds. Thus, whereas in the initial phase of the sprint at slow speeds and high ES, C_{aer} is a negligible fraction of the overall energy cost, this is not so at high speeds with ES tending to zero. Indeed, at the highest average forward speed ($\mathbf{v}_f$) attained in this study (9.46 m s^{-1}), C_{aer} amounted to about 20% of the total energy cost and required about 8 additional W kg^{-1} in terms of metabolic power. This is substantially equal to the data estimated by Arsac (2002) for sea level conditions.

Finally, the analysis presented in Fig. 1 shows that, in the deceleration phase, sprint running can be viewed as the analogue of downhill running at constant speed. According to Minetti et al. (2002), Eq. 13 can also be utilised to describe the energetics of downhill running. So, the negative values of ES, obtained when $\mathbf{a}_f$ is also negative, can be inserted into Eq. 14 to estimate the corresponding C_{sr} values in the deceleration phase. Quantitatively, however, the effects of deceleration on C_{sr} are much smaller than those described above for the acceleration phase, because throughout the whole range of the downhill slopes (from 0 to –0.45), the energy cost of running changes by a factor of 2, to a minimum of 1.75 J kg^{-1} m^{-1} at a slope of –0.20, rising again for steeper slopes, to attain a value about equal to that for level running (3.8 J kg^{-1} m^{-1}) at a slope of –0.45. This is to be compared with an increase of about fivefold from level running to +0.45 (see Eq. 13 and Minetti et al., 2002).

Conclusions

The above analysis and calculations allow us to condense the factors affecting the instantaneous energy cost of sprint running into one comprehensive formula:

$$C_{sr} = (155.4\text{ES}^5 - 30.4\text{ES}^4 - 43.3\text{ES}^3 + 46.3\text{ES}^2 + 19.5\text{ES} + 3.6)\text{EM} + k'\mathbf{v}^2, \quad (19)$$

where all terms have been previously defined. The corresponding metabolic power (P_{met}) is described by the product of Eq. 19 and the ground speed (**s**):

$$P_{met} = C_{sr} * \mathbf{s} = (155.4\text{ES}^5 - 30.4\text{ES}^4 - 43.3\text{ES}^3 + 46.3\text{ES}^2 + 19.5\text{ES} + 3.6)\text{EM}\mathbf{s} + k'\mathbf{v}^2\mathbf{s}. \quad (20)$$

When, as is often the case, the sprint occurs in calm air and hence **v=s**, these two equations can be easily solved at any point in time, provided that the time course of the ground speed is known.

List of symbols

A_{ns}	amount of energy derived from anaerobic stores
$\mathbf{a}_f$	forward acceleration
$\mathbf{g}'$	overall acceleration acting on the runner's body
C	energy cost
C_{aer}	energy spent against the air resistance per unit of distance
C_{sr}	energy cost of sprint running
d	distance
$\mathbf{g}$	acceleration of gravity
E_{tot}	overall energy expenditure
EM	equivalent normalised body mass
ES	equivalent slope
F	average force
F′	average force exerted by active muscles during the stride cycle
k'	proportionality constant
M_b	body mass
P_{met}	metabolic power
Pcr	phosphocreatine
s	ground speed
$\mathbf{s}_{max}$	maximal velocity reached during the sprint
t	time
t_e	performance time
v	speed
$\mathbf{v}_f$	forward speed
$\mathbf{v}_{max}$	maximal speed
V_{O_2}	oxygen consumption
x	incline angle between **g′** and the terrain
τ	time constant
η	efficiency

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