FLOW VISUALIZATION WITH STEREO SHADOWGRAPHS OF STRATIFIED FLUID

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SUMMARY

Water stratified thermally or with dissolved material has a refractive index that varies with height. The stratification has no obvious visual effect until the fluid is disturbed, for example by a fish, but then it marks the creature's wake with a jump in refractive index that is visible in shadowgraphic projection. Two shadowgraphs in stereo display the wake in three dimensions. For a permanent record, or for analysing events too quick for the eye to follow, movies can be taken of the shadowgraphs.

INTRODUCTION

Conventional techniques of flow visualization (Merzkirch, 1974) work poorly in much of biological fluid dynamics. Ink, smoke and most other markers have short lifetimes before diffusion makes them invisible or they settle out of the fluid. Hence engineers most often use them in water channels or wind tunnels that take them to the test model soon after they are made. If the model is a swimming fish, the channel inevitably limits his swimming repertoire and distorts the motions of his wake (Fig. 2) (McCutchcn, 1970).

Particles with nearly neutral buoyancy might appear to be the answer. They settle out so slowly that all the fluid can be marked and will stay marked until the creature does what he wants, wherever he wants to do it. But markers randomly distributed through the whole volume of fluid display the motions of thousands of individual points, which are hard for the brain to combine into a flow pattern. Let any doubter stir black pepper into water in a white container and see for himself.

In principle this inchoate mass of data could be recorded and then somehow boiled down to comprehensible form. In most practical uses of flow visualization, however, prior information allows the experimental data to be edited in advance. If the flow is time independent it can be explored in a succession of experiments, with only a single cross-section or a single streamline being marked in any one experiment. This approach could reveal the feeding currents of a sponge, but it is hopeless with a free-swimming animal like a fish.
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The fish swims in stratified water, hotter at top than bottom. Two point sources of light behind crossed polaroids project polarized shadowgraphs on the viewing screen. Each eye, behind its analysing polaroid sees one shadowgraph, and the two eyes together see a three-dimensional view of fish and wake floating in space.

THE METHOD

A method is needed that reveals flow anywhere in the fluid, yet presents its data in a well-edited form. The scheme described below does exactly this (Fig. 1). The water is thermally stratified, being hotter at the top than at the bottom. It appears homogeneous until disturbed by the fish. The waters that the fish parts with his nose will usually be out of register with each other vertically when they meet again at his tail. So in general there is a jump in temperature and refractive index across the surface of confluence between the waters from the two sides, which makes it visible in shadowgraphic projection as shown in Fig. 3 (McCutchen, 1975, see also Pierce, 1961; Deblcr & Fitzgerald, 1971). By showing the two eyes a pair of stereo projections the apparatus in Fig. 1 gives a three-dimensional view of the wake and of the fish.

The method shows nothing except the surface of confluence. But if the shape and motion of the surface of confluence are known the flow everywhere else can be calculated, at least with the approximation that the surface of confluence has zero thickness (Lamb, 1945). It seems that the human brain automatically does a rough calculation and infers the correct motion in the blank spaces. The data is thus edited in a hydrodynamically elegant way by removing all the redundancy, by paring it down to the smallest set that fully specifies the motion, a set that is easy for the mind to understand.

The method reveals surfaces of confluence however caused. If the fish puffs water out of his mouth the jet is quite obvious. A ‘cough’, perhaps the fish’s reaction to chlorine in the water, can launch a neat vortex ring.

Stratification has little effect on the flow if the hydrodynamic event lasts for much less than the Väisälä Brunt period:

$$2\pi \left[ g \left( \frac{d\ln \rho}{dh} \right) \right]^{\frac{1}{2}},$$
Fig. 2. A 10 cm trout swimming among ink streamers in a 10 cm wide water channel.

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(Facing p. 12)
Fig. 3. A 3.15 cm Zebra Danio swimming in the 'push and coast' mode, photographed with the apparatus shown in Fig. 6.

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where \( \rho \) is the density of the fluid and \( h \) is depth from the surface (Hill, 1962). For a temperature gradient of 1 °C/cm at 20 °C this is 14 s.

The tendency of the fluid to return to its original level, in conjunction with thermal conductivity, makes the patterns slowly fade. Fifteen seconds after one event the water is clear enough to record the next.

So long as the jump in refractive index across the surface is big enough to be seen in the shadowgraphs its magnitude is unimportant. The temperature gradient in the fluid must not be too small, but need not be uniform. One or 2 degrees C/cm gives adequate sensitivity over a depth of 5 or 10 cm without having any water too hot or too cold for comfort.

**PLUMBING**

To maintain the temperature gradient, I add hot water at the top, a litre or so per minute, cold water at the bottom and extract water at the middle. In a few minutes this produces a sharp temperature step at the level of the outlet. Stirring at the interface, by either the experimental subject or the experimenter, spreads this into a gradient.

So as not to disturb the water in the tank the hot and cold water should enter at low speed, for example, by being passed through sponges tied over the ends of the filling tubes. The extraction is conveniently done by either of the drains shown in Fig. 4 which remove water at mid level yet still regulate the depth of water in the tank.

If left unstirred, the stratification takes many minutes to die away, so continuous supply and removal of water are convenient but not necessary.
I use thermal stratification because it is cheap and convenient. Instead of cold water below there could be salt water, because salt, like coldness, raises both the density and refractive index of the water.

**THE MAGIC CAP: PROJECTION OPTICS**

Mounting the lights on a construction worker's hard hat, the 'magic cap' that makes water visible, is very convenient (Fig. 5). The analysers automatically stay aligned with the polarizers; the lights stay spaced apart in the same direction as the observer's eyes to preserve proper stereo conditions, and move with the observer's head to give normal parallax. The resulting view is natural except that it looks as it would if the observer had his eyes where the lights are.

At twelve o'clock relative to the shadowgraph of the fish lies a second, direct view of the fish, but one soon learns to ignore it.

As shown in the appendix, putting the lights about 10 cm apart, compared to the 6.5 cm spacing of the eyes, corrects the foreshortened appearance of objects immersed in a material of high refractive index. To increase the impression of depth, increase the spacing. Unless the lights are inconveniently located in front of the observer, the perspective in which either eye sees the object corresponds to an observer–object
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Arriflex 16 mm movie camera

Aquarium placed for horizontal projection

Field lens

Mirror

Field lens

Aquarium

Red filter and polaroid

Arc lamp

Diverging lens

Fig. 6. Apparatus for two-dimensional movies. The red filter minimizes the effects of chromatic aberration by the field lenses and the polaroid reduces unwanted reflections from the front surfaces of the mirrors.

distance greater than that given by the stereo convergence angle of the two eyes. However, the stereo clues seem to overrule the perspective clues, and the observer is unaware of any conflict.

The lights are 6 V microscope illuminator bulbs, run in series by an isolated 12 V storage battery to avoid any shock hazard.

VIEWING SCREENS

Any non-depolarizing screen will do. These include aluminium paint and matte finished metal. I prefer the matte side of household aluminium foil for its fine, uniform grain, and the fact that it spreads the reflected light more in the direction along the unrolled sheet than across it. With the long way parallel to the line between the eyes this evens the brightness of the two stereo views. I stretch the foil on a large, wooden embroidery frame. The water swells the wood and tightens the foil.

A positive plastic Fresnel lens laid on the viewing screen will bend light reflected near the periphery back toward the observer and brighten the otherwise under-illuminated edges of the view. But the observer must then keep his head nearly fixed in space, and I find the lens more trouble than it is worth.

MOVIES

The movie apparatus shown in Fig. 6 will take photographs for publication, or capture phenomena too quick for the eye to follow. Fig. 2 is part of a single frame from a movie taken of a 3.15 cm Zebra Danio (*Brachydanio rerio*) in an investigation of fish propulsion (McCutchen, 1975). The movies were used to measure the fish's Froude propulsive efficiency.
A fish swimming near the surface makes waves that distort the shadowgraph. These are easily prevented by a sheet of glass or transparent plastic floated on the surface with floats large enough to keep it from sinking, but too small to lift it clear of the water.

With two light sources side by side, stereo movies could be made with synchronized cameras or an arrangement to put both views on a single film.

SCHLIEREN?

Because it operates by bending the light paths, the shadowgraphic method causes some distortion, which sometimes misrepresents the shapes of small features in the flow. For taking photographs this can be avoided by changing to a schlieren system, but all the optics must then be of high quality. With shadowgraphs the field lenses and tank walls need only be good enough to keep the image of the point source smaller than the camera aperture.

MODEL EXPERIMENTS AND INSTRUCTION

The stratified fluid technique is about the only easy way to study the hydrodynamics of uncooperative living creatures. But even in experiments with models it is useful for its convenience and the way it presents only the essential data. During the fish investigation I experimented with models and with vortex rings of various initial shapes to learn about wake behaviour.

Abruptly squeezing a water-filled margarine tub with a hole in its lid makes a vortex ring. A non-circular hole makes a non-circular ring whose shape alters as it advances. Two or more holes make two or more rings that interact as they travel. Filling the tub with hot or cold water increases the visibility of the vortex cores within the rings.

With simple models one can demonstrate in a few minutes the breakdown of potential flow round a sphere started from rest, or, behind a wing, the vortex sheet that rolls up to form the starting and tip vortices and everywhere develops waves that turn into the random confusion of turbulence. The Karman trail shows a sinuous surface of confluence, rippling back from the cylinder that causes it like a flag behind a flagpole.

These are phenomena that anyone working in fluid dynamics should make and see for himself, so I suggest that the method has a place in teaching.

REFERENCES


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Fig. 7. Idealized ray paths. In most practical systems, such as Figs. 3 and 5, the eyes, the lights, and the centre of the tank are not coplanar. No origin is shown for the $x$ coordinate, because only differences between pairs of coordinate values are significant. The spans labelled $a$, $b$, $c$, $A$, $B$, and $C$ are the ranges over which $dx/n$ is integrated to calculate $a$, $b$, $c$, etc. Each of these integrals ($a$, $b$, $c$, etc.) equals the physical length of its range only when $n$ (refractive index) is unity everywhere within that range. The lamps are widely spaced to make the drawing clear. So spaced they would give an exaggerated impression of depth.

APPENDIX

The optical system is shown in Fig. 7. The walls of the tank are parallel to each other and to the screen. The axis of the system is perpendicular to these walls. The lamps lie on a line perpendicular to the optical axis, intersecting it at $x_L$. The fish and his wake we call the object. A typical axial point in the object is at $x_J$ (for object), the screen is at $x_S$, the reconstruction at $x_R$, the eyes at $x_E$.

The drawing shows the ray paths for the two lamps illuminating two points in the object, and the two eyes looking at the corresponding points in the reconstruction.

If the refractive indices of water and glass were the same as that of air, light would travel in straight lines, and all would be simple. But the refractive indices are different, and Snell's law requires that the sine of the angle between the optical axis and a particular light ray at any point be inversely proportional to the local refractive index $n$. In our case this angle is small enough so that its sine is not far different from its tangent. So if $y$ is the distance, measured in a plane perpendicular to the optical axis, between the points where two rays cut that plane, and $x$ is distance along the optical axis

$$\frac{dy}{dx} = \tan \alpha = \left(\tan \alpha_0\right)/n,$$

where $\alpha$ is the angle between the rays, and $\alpha_0$ is the angle between the rays anywhere that the refractive index is unity.
Integrating eq. (1) we have

\[ \Delta y_{x_1, x_8} = \int_{x_1}^{x_8} \tan \alpha \, dx = \tan \alpha_0 \int_{x_1}^{x_8} dx/n. \]  

(2)

If the lamps are a distance \( l \) apart, and the rays destined to shine on the same spot in the object set out at an angle \( \alpha_L \) to each other, then because the lamps are in air, \( \alpha_L \) is equivalent to \( \alpha_0 \) in eq. (2), which, if we set \( x_1 = x_L \) and \( x_8 = x_J \) becomes

\[ \Delta y_{x_L, x_J} = l = \tan \alpha_L \int_{x_L}^{x_J} dx/n = a \tan \alpha_L, \]  

(2a)

where for convenience we have set \( \int_{x_L}^{x_J} dx/n = a. \)

These rays continue on to the screen, which they meet at points a distance \( s \) apart, hence

\[ \Delta y_{x_J, x_8} = s = \tan \alpha_L \int_{x_J}^{x_8} dx/n = b \tan \alpha_L, \]  

(2b)

where

\[ b = \int_{x_J}^{x_8} dx/n. \]

If the eyes are a distance \( e \) apart and the rays which reach them from a single point in the reconstruction arrive at an angle \( \alpha_E \) to each other then

\[ \Delta y_{x_E, x_R} = e = \tan \alpha_E \int_{x_E}^{x_R} dx/n = A \tan \alpha_E, \]  

(2c)

where

\[ A = \int_{x_E}^{x_R} dx/n. \]

These rays came from spots on the screen spaced \( s \) apart, so

\[ \Delta y_{x_R, x_8} = s = \tan \alpha_E \int_{x_R}^{x_8} dx/n = B \tan \alpha_E, \]  

(2d)

where

\[ B = \int_{x_R}^{x_8} dx/n. \]

Combining Eq. (2a), (2b), (2c), and (2d) gives us

\[ lb/a = s = EB/A, \quad \text{or} \quad A/a = (eB)/(lb). \]

In terms of

\[ c = a + b = \int_{x_L}^{x_8} dx/n, \quad \text{and} \quad C = A + B = \int_{x_E}^{x_R} dx/n \]

this becomes

\[ A/a = e(C - A)/(c - a), \]

which we can solve for \( A \), getting

\[ A = \frac{ac}{(c - a)/e + a}. \]  

(3)
Because eq. (2c) shows that the rays approaching the eyes appear to come from a point at a distance \( A \), eq. (3) tells us how far away from the observer the reconstruction appears to be. Only if \( n = 1 \) everywhere on the path from reconstruction to observer will \( A \) also be the real distance \( x_R - x_g \), but for the observer the apparent distance is real and the real distance is unknowable! (In terms of a scale of distance defined by \( dX = dx/n \) the quantities \( a, b, c, A, B, \) and \( C \), would be true distances and the rays would be straight lines.)

The apparent size of the reconstruction transversely to the axis is the tangent of the angle \( \beta \) it subtends at one eye times its apparent distance \( A \) from the eye. From eq. 2 with \( \beta \) substituted for \( \alpha_a \) and \( x_g \) and \( x_R \) as the limits of integration, the apparent transverse size \( A \) tan \( \beta \) is just the real transverse size of the reconstruction, which we will call \( H_R \).

If the transverse size of the object is \( H_J \), the transverse magnification is \( H_R/H_J \). To get this in terms of the parameters of the system we note that if the size of the shadow on the screen is \( H_S \) then

\[
CH_R/A = H_S = cH_J/a, \quad \text{or} \quad H_R/H_J = cA/aC,
\]

from which we eliminate \( A \) with eq. (3) to get

\[
\text{Trans. Mag.} = \frac{H_R}{H_J} = \frac{c}{l(c-a)/e+a}.
\] (4)

The longitudinal magnification is the ratio between the distance that the reconstruction appears to move along the axis and the real axial motion of the object, so

\[
\text{Long. Mag.} = \frac{dA}{dx_J} = \frac{1/n_J}{dA/da},
\]

where \( n_J \) is the refractive index at the object, and the last equality holds because, from eq. (2a) \( dx_J = n_Jda \). Substituting for \( A \) the expression from eq. (3) we have

\[
\text{Long. Mag.} = \frac{cCl/e}{n_J[ll(c-a)/e+a]^3},
\] (5)

or, with eq. (4),

\[
\frac{\text{Long. Mag.}}{\text{Trans. Mag.}} = \frac{C}{n_J[C + a(e/l - 1)]}. \quad \text{(6)}
\]

The farther apart we put the lamps the smaller becomes the fraction \( e/l \) and the greater is the ratio of longitudinal to transverse magnification, hence the greater the apparent depth of the reconstruction.

The headlight arrangement is most convenient if \( c \approx C \). Setting them equal and taking the refractive index of water to be 4/3 we find that the longitudinal and transverse magnification are equal if

\[
e/l = 1 - c/4a.
\] (7)

The requirement can be met only if \( a > c/4 \). For lower values of \( a \) the longitudinal magnification will be deficient, but from eq. (6), so long as \( l > e \) it will always be at least \( 1/n_J \), or \( 3/4 \), of the transverse magnification.

For \( a > c/4 \) eq. (7) is satisfied by values of \( e/l \) that start at \( 0 \) for \( a = c/4 \) and rise to \( 3/4 \) as \( a \) approaches \( c \). In a typical experimental arrangement we might have \( a = 3c/4 \), which would make \( l = 3e/2 \), and put the lights about 10 cm apart.
Perspective

For the perspective to be natural the transverse magnification must be independent of $x_f$ and therefore of $a$. In the reconstruction of a cube, for example, the rear face must be the same size as the front face, hence from eq. (4)

$$d(\text{Trans. Mag.})/da = 0 = \frac{l/e - 1}{[l(c-a)/(e+a)]l}, \quad \text{so} \quad \frac{l}{e} = 1. \quad (8)$$

For natural perspective the lights must be spaced by the same distance as the eyes. From eq. (4) the transverse magnification is then unity. From eq. (6) the longitudinal magnification is also unity if

$$C = n_f c,$$

which in practical arrangements means that the lights must be nearer to the screen than is the observer.

From eq. (7) the headlight system with $c = C$ requires that $l/e > 4/3$. By eq. (8) $d(\text{Trans. Mag.})/da > 0$, the rear of the reconstruction will be larger than the front, and the perspective will look 'flat', i.e. it would be appropriate to an object at a greater distance.