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Constrained optimization in human walking: cost minimization and gait plasticity

John E. A. Bertram

Department of Nutrition, Food and Exercise Sciences, Florida State University, Tallahassee, FL 32306, USA

Present address: Department of Cell Biology and Anatomy, Faculty of Medicine, University of Calgary, Calgary, Alberta, Canada
(e-mail: jbertram@ucalgary.ca)

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Summary

As walking speed increases, consistent relationships emerge between the three determinant parameters of walking, speed, step frequency and step length. However, when step length or step frequency are predetermined rather than speed, different relationships are spontaneously selected. This result is expected if walking parameters are selected to optimize to an underlying objective function, known as the constrained optimization hypothesis. The most likely candidate for the objective function is metabolic cost per distance traveled, where the hypothesis predicts that the subject will minimize the cost of travel under a given gait constraint even if this requires an unusual step length and frequency combination. In the current study this is tested directly by measuring the walking behavior of subjects constrained systematically to determined speeds, step frequencies or step lengths and comparing behavior to predictions derived directly from minimization of measured metabolic cost. A metabolic cost surface in speed–frequency space is derived from metabolic rate for 10 subjects walking at 49 speed–frequency conditions. Optimization is predicted from the iso-energetic cost contours derived from this surface. Substantial congruence is found between the predicted and observed behavior using the cost of walking per unit distance. Although minimization of cost per distance appears to dominate walking control, certain notable differences from predicted behavior suggest that other factors must also be considered. The results of these studies provide a new perspective on the integration of walking cost with neuromuscular control, and provide a novel approach to the investigation of the control features involved in gait parameter selection.

Key words: gait, locomotion, metabolic cost, control, human.

Introduction

It has long been recognized that humans and other species use their limbs in a manner that will minimize the metabolic cost associated with locomotion (Elftman, 1966; Hoyt and Taylor, 1981; Zarrugh et al., 1974). It has been largely assumed that this minimization requires the system to be ‘tuned’ to a specific functional relationship, such that the relation between basic gait parameters, speed, step frequency and step length, results in a consistent minimization function (Fig. 1A; Grieve, 1968; Anderson and Pandy, 2001). That is, if energetic cost minimization for a given speed requires a specific combination of step length and frequency, then it likely implies that cost minimization for that particular step length should require the selection of the same combination of frequency and speed. We have recently demonstrated that this is only the case for the conditions that determine the absolute minimum cost (Bertram and Ruina, 2001). At any speed other than the preferred walking speed (at which cost is least), independent constraint of speed, step frequency or step length results in three different speed–frequency (or speed–step length or frequency–step length) relations (Fig. 1B). The step length constraint relationship differs substantially from the other two, but all three relations differ significantly (Bertram and Ruina, 2001).

To account for this observation we have proposed the constrained optimization hypothesis (Bertram and Ruina, 2001; Fig. 2). Walking parameters (such as speed, step frequency or step length) will be selected to minimize an underlying objective function for the constraints imposed on the system. The combination of gait parameters that provide the minimization will depend on the criteria that constrain the function, i.e. what feature of walking is the end objective. This implies that a variety of combinations of speed, step length or frequency can be selected, depending only on the conditions externally imposed on the individual. Thus, the human walking system is expected to display substantial behavioral plasticity, determined by the circumstances under which it is operating and largely unconstrained by higher order control features built into the coordination system (Holt, 1996).

The above implies an integration between neuromuscular function and the physical circumstances that ultimately determine the most appropriate behavioral features of the gait. The constrained optimization hypothesis, if validated, would provide a new tool for investigating this integration. It provides a theoretical framework with specific predictive expectations that can be tested against hypotheses designed to evaluate and identify the limits and precision of the integration, and provides a means to explore the underlying mechanisms responsible. Assuming that human walking responds to other environmental constraints in a manner similar to that of the fundamental gait-determining parameters such as speed, step length and frequency, full understanding of constrained optimization might also provide a more precise means of predicting locomotion behavior under a range of normal and abnormal circumstances.

Behavioral data from a range of subjects fit well with the expectations of the constrained optimization hypothesis (Bertram and Ruina, 2001). In fact, the response is so predictable that this experiment can be replicated as an undergraduate laboratory exercise (Bertram, 2002). However, two important questions remain: what is the nature of the objective function that determines the optimization, and what are the limits of its influence?

Before the second question can be approached, the first must be determined. It has been suggested that the most obvious candidate for the objective function is the metabolic cost of locomotion, and there is some circumstantial evidence supporting this view (Anderson and Pandy, 2001; Bertram and Ruina, 2001; Kuo, 2001). It is the purpose of this study to evaluate the metabolic cost of locomotion as the objective function of constrained optimization in human walking. This is accomplished in three steps: (1) generate a map of the metabolic cost function for all combinations of walking parameters for a group of healthy subjects and plot this in speed–step frequency space (a metabolic cost surface), (2) use this empirical function to predict the optimum walking gait parameters under specific imposed walking conditions using the constrained optimization hypothesis as a predictive model and (3) compare the predicted behavior to the behavior independently selected by the subject group under the specified constraint conditions.

The above experiment constitutes a direct test of the constrained optimization hypothesis using metabolic cost of travel as the objective function. Association of the observed and predicted behaviors will indicate that metabolic cost is a substantial consideration in the automatic selection of basic parameters in human walking, even though the selected parameters may not match those routinely used in walking.

Methods

Subjects

Ten subjects were tested (four male and six female, ranging in age from 20 to 30 years). Information on basic physical characteristics of each subject is provided in Table 1.

All procedures performed were approved by the Florida State University Human Subjects Institutional Review Board and approved informed consent forms were acquired from each subject prior to participation.

Table 1. *Characteristics of the subjects studied*

Subject	Sex	Age (years)	Mass (kg)	Leg (m)	Height (m)
S1A	F	23	59.4	0.81	1.58
S3C	F	20	67.0	0.925	1.75
S4B	M	21	66.4	0.88	1.68
S6B	F	20	58.2	0.865	1.68
S8B	M	29	74.5	0.91	1.74
S9C	M	22	79.1	0.885	1.75
S10A	F	22	65.9	0.925	1.73
S11A	F	30	53.4	0.825	1.62
S12B	M	25	81.6	0.973	1.82
S14C	F	20	54.9	0.838	1.64

14 subjects (7 male, 7 female) started the series of experiments; listed are those who completed all aspects of the project. Within the subject label the designation A, B or C refers to the time of day that the metabolic measurements were made: A, morning; B, afternoon; C, evening. Leg refers to the length of the pelvic limb while standing erect, measured from floor to lateral aspect of the greater trochanter.

Metabolic cost measurement

The metabolic cost of walking was determined for each subject by measuring O_2 consumption and CO_2 elimination rates when walking on a treadmill at a set speed while the subject matched their step frequency to the audible tone of an electronic metronome. Note that constraint of both speed and step frequency also implies constraint of step length, because only a single step length will provide the appropriate speed for a given frequency. For each subject, 49 metabolic measurements were taken over a range of speeds and frequencies that covered the majority of the range physically possible. Table 2 illustrates the treadmill speeds, step frequencies, the sequence of speed–frequency combinations used and mean energy expenditure (J kg^{-1} m^{-1}) for the subject group.

Each subject participated in four metabolic cost measurement sessions, each running on a different day but all occurring within one week. The first session was used to familiarize the subject with the protocol of the study, the gas analysis equipment (including mouthpiece, nose clip and supporting head gear) and to practice walking on a treadmill in time with a metronome beat. Each of the following three sessions were used to evaluate metabolic cost of walking, with the 49 measurements distributed between the three measurement sessions. All metabolic measurements were taken at least 2 h postprandial. A baseline metabolic rate was taken at the beginning and end of each measurement session, determined as the mean of the average over the last 2 min of a 10 min period of quiet standing. This served two roles. Consumption rate before and after the session was checked for equivalence. Metabolic rate at the end of each measurement session was within 3% of the original (and was usually indistinguishable from the original value), indicating that the procedure did not overtax the subjects. The mean of the beginning and ending daily baseline was also used to adjust for occasional small metabolic rate differences on different measurement days.

Table 2. *Speed and frequency combinations used to characterize the metabolic cost of walking*

Speed ($m\ s^{-1}$)								
2.34						4.35 ± 0.41 **20**	4.16 ± 0.53 **39**	4.18 ± 0.50 **21**
2.07					3.85 ± 0.52 **23**	3.27 ± 0.47 **27**	3.42 ± 0.46 **22**	3.69 ± 0.39 **49**
1.78				3.72 ± 0.60 **24**	2.38 ± 0.34 **44**	2.59 ± 0.30 **25**	3.07 ± 0.54 **38**	3.34 ± 0.41 **26**
1.5			5.19 ± 0.78 **32**	2.45 ± 0.29 **11**	2.08 ± 0.24 **43**	2.36 ± 0.39 **10**	2.83 ± 0.36 **37**	3.43 ± 0.78 **9**
1.16		4.96 ± 0.81 **19**	2.90 ± 0.31 **31**	1.94 ± 0.32 **18**	1.92 ± 0.31 **42**	2.49 ± 0.35 **17**	3.11 ± 0.55 **36**	3.59 ± 0.73 **16**
0.86	6.51 ± 1.40 **5**	2.94 ± 0.52 **45**	2.14 ± 0.31 **6**	1.89 ± 0.27 **33**	2.37 ± 0.40 **7**	2.75 ± 0.58 **28**	3.44 ± 0.49 **8**	4.01 ± 0.79 **48**
0.56	3.27 ± 1.02 **40**	2.35 ± 0.38 **4**	2.22 ± 0.60 **30**	2.58 ± 0.47 **3**	2.99 ± 0.63 **41**	3.58 ± 0.47 **2**	4.24 ± 1.04 **35**	4.87 ± 0.98 **1**
0.26	3.61 ± 1.10 **12**	3.54 ± 1.00 **46**	4.28 ± 0.59 **13**	4.55 ± 0.97 **34**	5.85 ± 0.89 **14**	6.11 ± 1.48 **29**	7.70 ± 1.27 **15**	8.22 ± 1.66 **47**
	0.8	1.1	1.33	1.11	2.1	2.4	2.67	2.93

Frequency (steps s^{-1})

Mean metabolic cost ($J\ kg^{-1}\ m^{-1} \pm$ S.D.) from the subject pool (N=10) for each combination are listed in the center of each cell. The bold number in each cell lists an example measurement sequence where cells 1–16 were measured on the first data collection day, 17–31 on the second day and 32–49 on the final day. The measurement sequence was chosen to minimize the chance of order effect (see text for details). The unfilled cells indicate speed–frequency combinations that are extremely difficult to perform using a walking gait.

During each of the three walking cost measurement sessions, O_2 consumption and CO_2 elimination were monitored from 16–17 5 min activity bouts in which speed and step frequency were controlled. Rest periods of at least 2 min were allowed between activity bouts to reduce the effects of fatigue. The subjects were encouraged to take more time if they felt short of breath or fatigued. The O_2 consumption and CO_2 elimination rates were taken as the mean of the last 2 min of the 5 min walking trial. The exercise duration was purposely kept as brief as possible so that more measurement points could be collected in each session. It is recognized that the 5 min collection period, with the initial 3 min constituting the transition to steady consumption rate, is marginal. Oxygen consumption and CO_2 elimination rates were visually monitored following the 3 min point and the measurement period was increased by one or more minutes if a convincing steady state had not been achieved by the beginning of the fourth minute. Note that almost all of the walking trials used in this analysis were not aerobically challenging. The subjects were not rigorously trained athletes but all were active individuals with good fitness levels and many subjects did not require data collection extensions for any of their speed–frequency combinations. Oxygen consumption and CO_2 elimination rates (STP) were determined using a commercially available metabolic analysis system (TrueMax 2400, ParvoMedics, Salt Lake City, UT, USA).

The sequence of speed–frequency combinations followed a psuedo-randomized order. For each subject the complete grid of speed–frequency combinations was pre-determined and divided into rows and columns. The rows and columns were numbered in alternate sequence. The three sessions in which measurements were made were divided into 16 or 17 speed–frequency combinations each (for a total of 49 measurements over three sessions). The order of speeds and frequencies for a given subject was determined by randomly selecting the number for the row or column. Once a row or column was selected, metabolic measurements were made for the speed and frequency of every second cell. Rows and columns were randomly selected until all cells were measured. This reduced the chance of an order effect in the measurement of the metabolic data. The order for one subject is listed in Table 2 by the numbers in the lower right corner of each cell (1 through 49).

Oxygen and CO_2 exchange rates were converted to caloric equivalents (Lusk, 1924), according to the assumption of low-

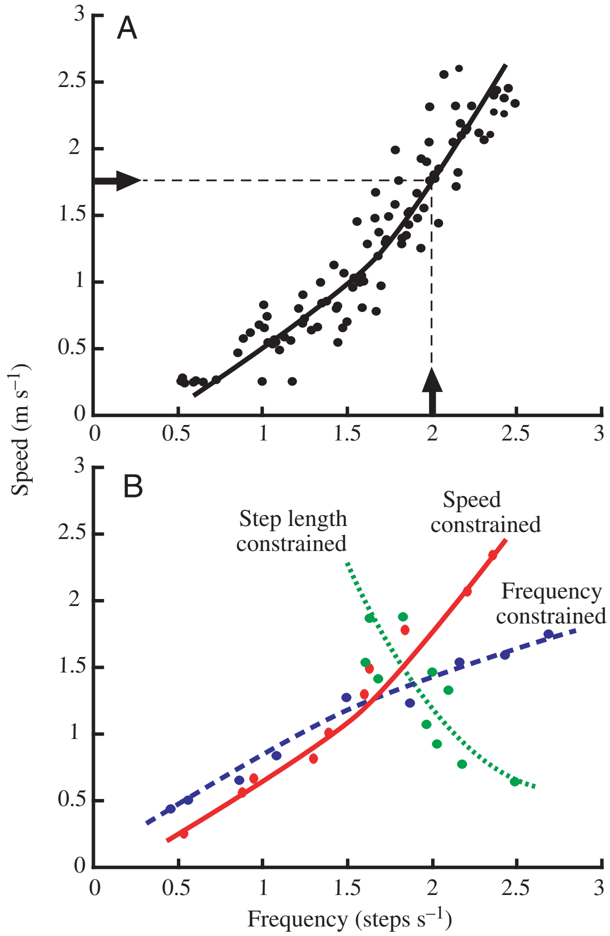

Fig. 1. (A) The relation between speed and step frequency in human treadmill walking (data from a mixed gender group of college-age students participating in an undergraduate biomechanics class). A consistent relationship is evident and is usually explained as indicating the most economical means of walking. If this is the case, then it should not matter if speed is determined, as in treadmill walking, or frequency is determined, as walking to a metronome beat (arrows). (B) Different constraints on walking actually result in different speed–step frequency relations (data from one subject). Red, solid line: speed constrained walking (treadmill); blue, broken line: frequency constrained walking (metronome); green, dotted line: step length constrained walking (floor markers).

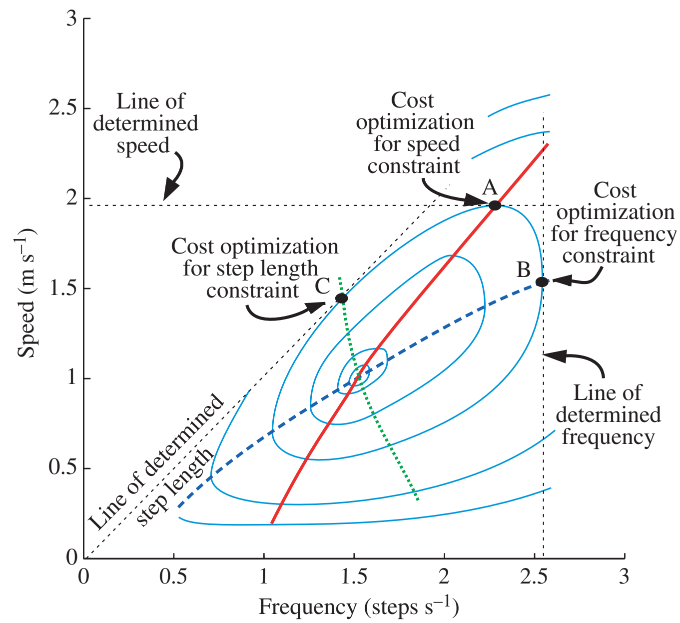

Fig. 2. The constrained optimization hypothesis. Light blue lines indicate constant cost contours describing the metabolic cost surface for walking, where each contour indicates a higher cost than the one inside (central contour surrounds the minimum cost region). These are generalized contours, roughly following measurements of Molen et al. (1972). For each constraint (speed, frequency or step length), walking cost will be minimized if the individual selects the other gait parameters that occur where the line of determination just touches a cost contour; at any other point the cost will be greater for that constraint. Constraints indicated by line colors and form as in Fig. 1B.

to-moderate intensity work load and a respiratory exchange ratio (RER) below the blood CO_2 buffer level (Romijn et al., 1992). Metabolic cost measurements were considered acceptable if the average RER value over the final 2 min of each trial was 0.92 or less. Caloric cost rate was converted to Joules s^{-1} and normalized for the mass of the individual. This metabolic cost rate was then normalized as a metabolic cost of travel by converting the consumption rate to energy per distance traveled, as determined by the treadmill belt speed. Belt speed was directly measured for each trial by hand timing belt marker progression while the subject was moving on the belt.

Self-selected walking behavior

Walking behavior under each of the three walking constraints was determined for each subject on a day distinct from those in which metabolic measurements were made. Three constraints were applied: walking at a set speed (on a treadmill), walking at a set frequency (level walking to a metronome beat) and walking using a set step length (level walking matching step length to evenly spaced floor markers). The subject's response was determined by measuring the selection of the remaining two variables in the relation $v=df$, where v=forward speed, d=step length and f=step frequency. In each condition the subjects were told simply to walk in a manner that felt comfortable under the imposed conditions. This part of the study matched the procedures described in Bertram and Ruina (2001), which are briefly reiterated below.

Walking at constant v was controlled using a treadmill (Woodway, Desmo Pro, Waukeshaw, WI, USA) set at constant belt speed. Ten belt speeds were used, covering a range both above and below the subject's preferred walking speed (from 0.258 to 2.34 m s^{-1}). Not all subjects were able to walk at 2.34 m s^{-1}, so these subjects were measured at nine speeds. Step frequency at each v was measured by timing the duration of two sets of 20 steps after at least 1 min of walking at that v. Times for the two sets were averaged. Since v was set as treadmill belt speed and f was directly measured, step length could be calculated as $d=v/f$.

Constant f was imposed by asking the subject to walk to an

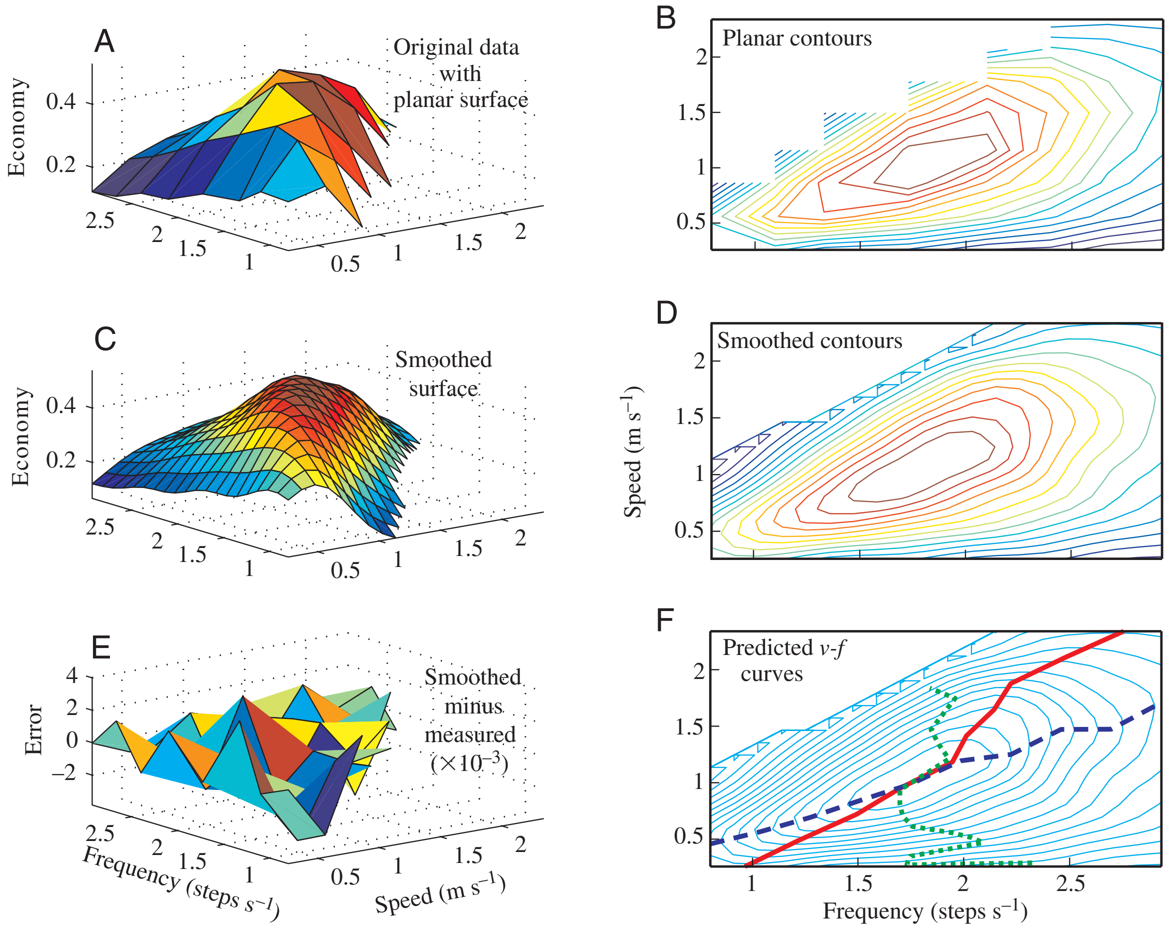

Fig. 3. An illustration of the process for determining the metabolic cost contours and generating the optimization predictions for walking. The pooled data for the subject group (490 original points) are shown. (A) The 49 metabolic cost measurements do not provide an adequate characterization of the cost surface and they generate contours without predictive value (B). Here and in C the cost is plotted as Economy, the inverse of cost, to allow better visualization of the surface. (C) The data were smoothed and the region between measured data points was reconstructed by fitting with a plate using a defined number of Fourier terms as the model. (D) From the smoothed surface, realistic contours are derived. (E) Error resulting from the fitting process can be monitored. (F) Predictions of speed and step frequency according to the constrained optimization hypothesis (as in Fig. 2) can be derived from the smoothed contours. Line colors and form for each constraint match those of previous figures.

electronic metronome beat. Frequencies were both above and below the subject's preferred step frequency. Ten frequencies were selected at intervals between 0.80 and 2.93 steps s^{-1}. The subject walked on a level corridor and was timed over a measured distance twice during the circuit. The two measured v values were then averaged. Timing was accomplished using a stop watch, but this was facilitated by displaying video images of the beginning and end points of the measured distance for the stationary timer as the subject passed. In this case f was set, v was measured and step length again calculated as $d=v/f$.

Constant d was imposed by asking subjects to step in registry with tape markers arranged at even distances on a level hallway over a 30 m length. Ten marker spacings were used, ranging from 0.236 to 1.01 m (normal step length for an average individual is approximately 0.56 m). For each d, presented to each subject in a different random order, the duration of 20 steps was timed. This measurement was made for each of two runs per individual and the average of the two was taken. In this case d was set, f was measured and speed was calculated as $v=fd$.

Analysis

The most challenging aspect of the analysis is the characterization of the metabolic cost contours, from which predictions of minimum cost walking could be derived. Metabolic cost data were pooled within the subject group and the cost surface was estimated from means at the 49 measured speed–frequency combinations.

The wide range of speed and step frequency combinations for human walking mean that even with 49 metabolic cost data points this represents only a sparse characterization of the cost surface (Fig. 3A). Such a characterization does not provide the resolution necessary to reasonably predict the selection of walking parameters directly from the raw measurements (Fig. 3B). In order to estimate the metabolic cost surface from the data, it was necessary to generate a continuous cost surface between the measured data points (interpolate). To accomplish this we use a custom-designed plate-fitting routine written in Matlab® (The Mathworks Inc., Natick, MA, USA).

The analysis routine is an approximate mathematical realization of a physical idea that may be described as follows. Say we have a few measured points representing a function of two independent variables. Plotting the independent variables along x and y directions, and the measured function value along the z direction, we get a set of points in three dimensional space. These points lie on, or (allowing for measurement error) close to, some curved surface. Our aim is to reconstruct that surface. The physical idea is to imagine a large, thin, near-horizontal plate with finite bending stiffness. Imagine attaching each measured (x, y, z) data point to the corresponding (x, y) point on the plate with a very short, stiff spring. Each spring tries to move its attachment point on the plate close to its corresponding measured data point (where the data point is considered fixed in parameter space) but the properties of the plate resist bending. The net result is that the plate takes on a smoothly deformed (bent) shape as it passes somewhat close to each anchored data point. If we use stiffer springs, then the fit at the data points is better; however, softer springs give a smoother overall surface. The stiffness of the spring thus provides a user-adjustable parameter for the surface-fitting process. This data-fitting approach, and variants thereof, are widely used. For example, the well known Bezier surfaces employed in many computer-assisted drawing applications use this basic idea (Foley et al., 1990). The approximations in the realization involve constraining the plate at its edges, which are taken rather far from any of the data points; and also in describing the deflected shape itself using a truncated double Fourier sine series. Such functions are routinely employed in a number of surface-fitting applications (Brown and Churchill, 2001). Examples include applied mathematics (Yee, 1981), climate modeling (Cheong, 2000a,b) and material analysis (Winterbottom and Gjostein, 1966, Saylor et al., 2000).

The mathematical approximations of the metabolic cost surface used in the current analysis may be viewed as arbitrary but useful simplifications that do not affect the quality of the surface fit (as is borne out by our graphical results and the modest error measures generated). We believe, moreover, that any other reasonable fitting process would give essentially the same results, since it is simply an issue of creating a continuous surface guided by the location of the measured data, which are relatively numerous and evenly distributed over the entire region of concern.

Once a smooth surface is fit to the collected data (Fig. 3C), contours can be generated (Fig. 3D). Fit is evaluated by comparing the original data to the surface estimate (Fig. 3E). Speed *vs* frequency relations for the three constraint conditions can be predicted from the cost contours as described in Fig. 2 (Fig. 3F).

The fit in the analysis used here has two control characteristics. First, the stiffness matrix, which determines the resistance of the plate to diverge from the data points (the stiffness of the spring analogy described above). The second control parameter sets the number of Fourier nodes, which determines the complexity of the Fourier models that generate the surface function. The fewer the nodes the more basic the shape of the surface. Although the interpolation procedure is non-standard, it suited the purposes of this study as the evaluation strategy was developed. Currently other strategies are available that generate similar surface predictions; nonuniform rational B-splines (Chaturvedi and Piegl, 1996) accomplish the same task and are gaining in popularity.

As indicated by the raw contours derived from simple prismatic connection of the data shown in Fig. 3B, some smoothing and interpolation of the cost surface between measured points was necessary to generate usable gait predictions. Not having a previous convention to work from, the strategy in this study was to employ a modest degree of smoothing and modify the data as little as possible. The parameters set in the analysis used in this study are shown in Fig. 4D–F where 14^2 (196) nodes are used with a stiffness matrix of 10^5. The smoothing function with these control parameters can be compared to that generated by equivalent nodes and a more pliant stiffness matrix of 10^3 (Fig. 4A–C) or to 4^2 (16) nodes with an equivalent stiffness matrix of 10^5 (Fig. 4G–I). The plate-fitting function control parameters used in the current analysis generate modest error values (note in Fig. 4F that this error landscape is an order of magnitude less than the other examples).

The gait parameter combinations predicted by constrained optimization from the metabolic cost contours are quite specific. The smoothing strategy left as much information content as possible in the surface. However, this leaves the specific prediction vulnerable to minor irregularities of the surface, and the 'optimum' thus determined may jump in unlikely ways (see for instance the step length constrained prediction, green dotted line, in Fig. 3F). If walk parameter selection is guided not so much by absolute optima but instead by limits, i.e. a range beyond which it is disadvantageous to operate, then it is best to evaluate the predictability of the cost function from its shape in the region of the optimum. The optimum is determined as the tangent to the cost contour for each parameter (Fig. 2). The curvature of the cost contour in the region of the tangent determines the specificity of the optimization. That is, a highly curved contour will require the selection of gait parameters that closely satisfy the optimization requirement. Under these circumstances deviation from the tangent would incur substantial extra costs. However, in the region of a straight or gradually curving contour, the selection of gait parameters that differ slightly from the absolute optimization will not have such large cost implications. In Fig. 5 the optima predicted by the constrained optimization criteria are indicated by ranges around the optima. This is done to provide a sense of the influence of variation in the contour slope at the tangent point. A series of ranges are shown as shaded regions where the deviations represent ±1% (red), ±5% (orange), ±10% (yellow), ±15% (gray) differences from the slope at the tangent point (Fig. 5).

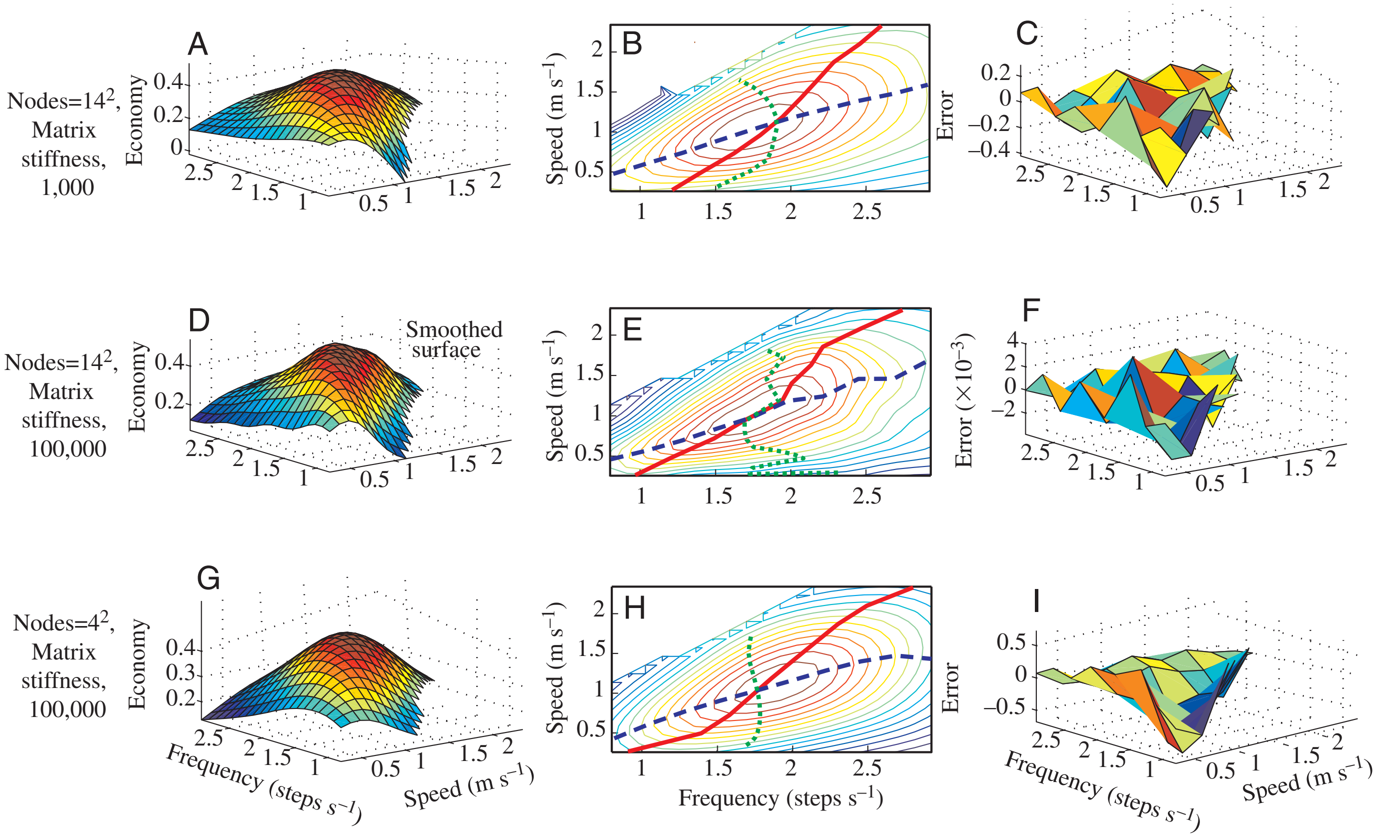

Fig. 4. Examples demonstrating the effect of the surface modeling control parameters on the surface, contours and error for the pooled data of this study. The two control parameters are matrix stiffness and number of nodes. In each series of plots the control settings listed at the left are used to generate each of the plots in line to the right. Matrix stiffness refers to the resistance of the smoothing function to allow deviations from the measured data (at the 49 measured points on which the surface is based). The higher this value, the greater influence local points have on the plate form. This can be seen by comparing A and D, where the number of nodes are kept constant and the influence of high stiffness matrix is evident. The number of nodes refers to the elements included in the truncated double Fourier sine series that generates the curvilinear model of the surface. In this case, the greater number of nodes, the more complex the surface shape can be. The influence of this parameter can be seen by comparing surfaces D and G. The analyses performed in the present study used the parameter settings indicated by plots D–F. These maintained much of the complexity of the measured data but linked those points in a more realistic manner than simplistic planar surfaces (Fig. 3A,B). This is indicated by the modest error values (difference between surface model and measured data) for the parameter settings used (error axis resolution in F is an order of magnitude greater than in C or I). For comparison of the optimization predicted for each parameter setting, the optimizations are provided (as in Fig. 3F). General predictions are consistent even over the wide range of control parameters illustrated, but the excessive smoothing of the extreme options removes all subtle features of the surface and increases overall error.

Data display and regression analysis

Self-selected walking behavior and optimization predictions are most conveniently compared between different constraint conditions when plotted on identical axes. However, this means that the independent variable may not be represented on the abscissa as least-squares regression technique demands. When necessary for plot comparisons, the regression equation was determined using the appropriate independent variable, but the variables were then mathematically replaced to represent the arrangement on the comparison plot. For example, the data of Fig. 5A were generated using speed as the controlled (independent) variable. However, for comparison to the other constraint conditions they are plotted with frequency on the abscissa. The regression line depicted was generated using a least-squares power function from a plot with speed as the abscissa, then the equation so determined was plotted against the transposed axes in Fig. 5A. This regression differed substantially from that which would have been generated using the variables as plotted, i.e. with frequency treated as the independent variable for all constraints. Such manipulation was done whether the independent variable is represented on the plot but not as the abscissa (as in Fig. 5A), or whether the independent variable was simply inferred by the plot but was not actually represented (as in Fig. 5C). Inspection indicates that the regressions provide a reasonable representation of the mean behavior of the subject group. Note that the use of a power function to fit some of these data is not intended to imply that a power function determines the relationship between these variables. Rather, it is a curvilinear relationship that appears to fit the data and is also convenient to manipulate, as described above.

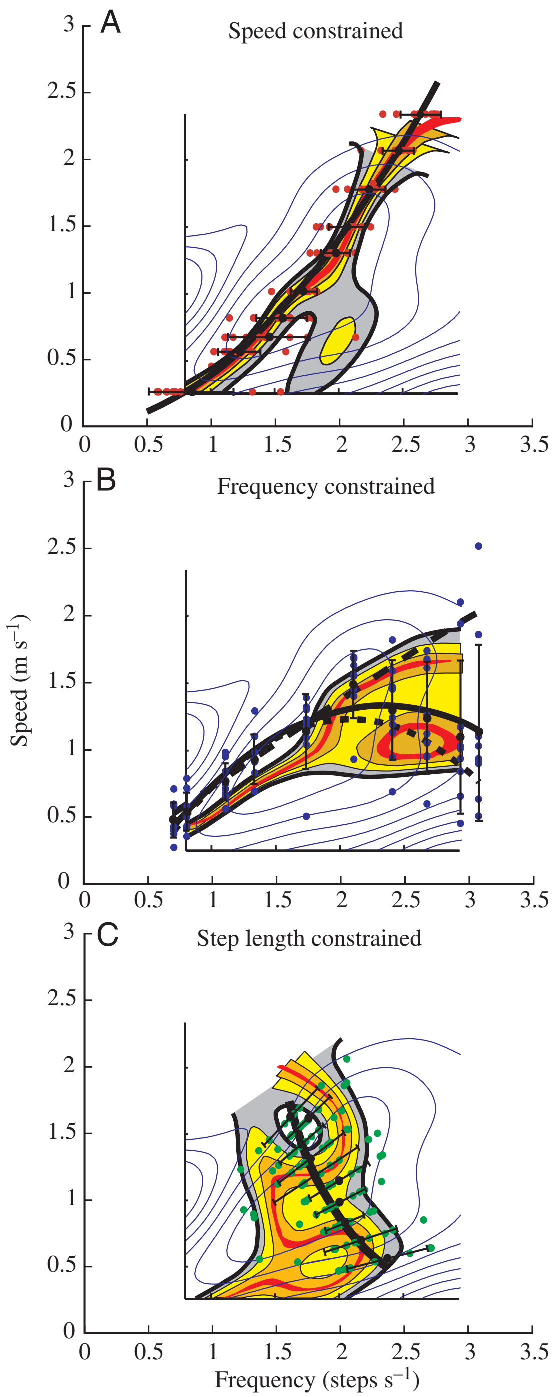

Fig. 5. Results of spontaneous gait selection parameters (data points) compared to minimum metabolic cost as predicted by the constrained optimization hypothesis. In each case the predictions are shown as a range around the tangent point indicating cost minimization for that constraint, where red indicates ±1%, orange ±5%, yellow ±10% and gray bounded by black ±15%. Mean for each constraint is indicated by a solid black circle with error bars showing ± 1 S.D. The data points for speed and step length constraints are fit with a power function (solid black line) with the constraint as the determinant variable (see text for details). For the frequency constraint (B) regressions of those subjects that chose the higher speed (long dashes) and lower speed (short dashes) options are also indicated. For all frequency constraint regressions a simple quadratic was used. Metabolic cost contours used to derive the constrained optimization predictions are shown in blue for reference. Data points are plotted in colors associated with the particular constraint applied, as in previous figures.

Results

The predictive value of the constrained optimization hypothesis is evaluated by plotting the predicted speed–frequency ranges for each constraint condition with the speed–frequency behavior displayed by the subject population walking under those specific constraints (Fig. 5). As described in Fig. 2, the optimizations are determined by the slope of the cost contours relative to the applied constraint; as plotted in Fig. 2, speed constraint is a horizontal tangent, frequency constraint is a vertical tangent and step length constraint is a tangent to slopes emanating from the origin. Note that this display is reversed from that used in Bertram and Ruina (2001); the current convention has the advantage of indicating longer step lengths as steeper slopes. The constrained optimization predictions are plotted as optimization ranges, where each range indicates a percentage difference in slope of the contour in the vicinity of the optimization (tangency point). Some cost contours are also shown to indicate the general shape of the cost function. For convenience of comparison, all data are arbitrarily but consistently plotted as speed ($\mathrm{m\,s^{-1}}$) against frequency (steps $\mathrm{s^{-1}}$); a similar analysis could have been performed using any two of speed, step frequency or step length.

First, observe that the constraint manipulations that make up this experiment are evident from these data. In the speed constrained condition the data points lie at discrete speed levels (Fig. 5A), as expected for the pre-selected treadmill speeds used as the control manipulation in this case. Inter-subject variability at each predetermined speed is expressed as variability in step frequency (and therefore also step length). Likewise, in the frequency constrained data the variability within each discrete frequency is expressed as a range of selected speeds (Fig. 5B). Slightly less obvious are the discrete step lengths in the step length constrained plot. Since $v=df$, on this plot a constant step length is indicated as a slope emanating from the origin, where greater slopes indicate longer step lengths and lesser slopes shorter steps (Fig. 5C). In this latter case inter-subject variability is expressed as a combination of speed and frequency, but that variability is limited to a constant slope. Attempts to reduce the data variability within any of the constraint conditions using standard normalization techniques based on conversion to non-dimensional terms (i.e. based on speed and frequency variants of the Froude number) did not substantially reduce the inter-subject variability. The data are presented in the absolute form as the most direct illustration of the experiment and its results.

The subject group was purposely selected to represent a wide range of body forms and sizes and thus is expected to indicate a general test of the constrained optimization hypothesis. However, this diversity resulted in substantial scatter of the behavioral data. The inherent variability presents a challenge when evaluating the association between the self-selected walking parameters and those predicted by constrained optimization of walking cost. In order to represent appropriately the behavior selected for each speed, frequency and step length constraint, the mean and standard deviation for the group are plotted (Fig. 5). The speed and step length constrained walking behavior data have been fit with a power-function regression. Some of the frequency constrained walking behavior data are not well represented by a power function. These data are fit with quadratic regressions, using either the entire data set (solid line), the subset that chose the higher speed option (long dashes) and the lower speed subset (short dashes).

For speed constraint conditions (Fig. 5A) the data correspond well with the ±1% optimality. The ±5% optimality zone includes all mean values and a majority of the standard deviations and clearly represents the behavior data. Greater scatter of the data is noted at extremely slow speeds.

For frequency constraint conditions the range of all optimality zones are highly restricted up to frequencies of 1.8 Hz. Below this frequency the means of the behavior data follow the trend indicated by the optimum, but are displaced toward higher speeds than predicted (by approximately 1 standard deviation). At higher step frequencies, cost optimization predicts a wide range of cost-effective speeds, including multiple optima for each frequency. The behavioral data display a substantial degree of variability in this area as well. At higher frequencies the subject population displayed two behavioral strategies: one that appeared to fit the higher speed optimization, and one that matched the lower speed optimization. Although substantial scatter exists, speeds between the two optima were not selected.

For step length constraint conditions, ±1% and ±5% optimality winds back and forth through the major distribution of the behavior data. The ±5% optimality zone includes six of the ten means. Three of the means outside the prediction range lie within an area of the cost optimization zone bounded on both sides by more optimum areas. The regions in which these means lie do not vary much from the optimization criteria. Many of the standard deviations reach beyond the optimization zone. This is particularly true for the right boundary where many selected speeds and step frequencies exceed those predicted.

In order to allow tracking of individual responses to the imposed constraints, the data for each subject are provided in Fig. 6. Female and male subjects are separated, as are specific constraint conditions, but each individual is labeled independently within the gender. These are the same data plotted as in Fig. 5.

Discussion

Test validity

The walking behavior of the study group measured under the applied constraint conditions match those observed by Bertram and Ruina (2001). In the previous study it was found that, within an individual, the walking speed–frequency relationships for each of the constraints usually differed significantly from the others. The present data indicate that these trends hold for grouped data as well. This verifies that human subjects respond to each of these imposed constraints in a distinct, specific and reasonably reproducible manner.

What are the potential explanations of this result? One possibility is that the observed responses that differ from normal, speed constrained walking simply represent neurologically determined artifacts, where the observed relationships indicate unnatural expressions of walking in the peculiar circumstances forced upon the subject. Such neurological factors could be psychological in origin, where such things as attention to foot placement to match floor markings is the source of interference with normal gait, or more functional interference with the neural control system itself, such as central pattern generator (CPG) coordination. In either case, the suggestion is that imposed constraints disrupt the normal gait pattern by interfering with the natural control system. This is most easily imagined with the step length constraint, where the overall speed–frequency relationship (Fig. 5C) differs markedly from what we consider 'normal' (i.e. speed constrained walking, Fig. 5A). This may suggest that something unnatural is indeed occurring in the system. In normal walking, both step length and frequency increase to achieve increases in speed. In contrast, when walking is constrained to set step lengths, the tendency is for step frequencies to remain constant or decrease slightly as step lengths and speeds increase (Fig. 5C).

Rather than indicating an artifact, however, the relationship found for step length constraint is consistent with both the constrained optimization of cost (the prediction from the measured cost contours, Fig. 2) and is to be expected from simple mechanical considerations of walking dynamics for this constraint. Consider the objective of step length constrained walking. In its purest sense, the goal would be to move the foot (and as a consequence, the center of mass of the body) from marker to marker for the least metabolic cost. In this case the objective of the task becomes dominated by the mechanics of the swing limb because it is placement of the foot near the marker that is the main objective of the step, with no predetermined requirement for the details of progress achieved. It is well recognized that the swing limb, though complex, ultimately has pendular characteristics (Mochon and McMahon, 1980a,b; McGeer, 1990; Garcia et al., 1999). One fundamental feature of a pendulum is that the period is approximately constant regardless of swing excursion. Thus, one might assume that if the body uses the natural pendular frequency characteristics of the swing limb as one component of a cost minimization strategy, then under constrained step length a constant 'natural' frequency would be employed. Certainly from Fig. 5C, it can be seen that a single step frequency (2 steps s^{-1}) is used by one or more subjects at all step lengths. In spite of the wide range of morphology in the subject group (body mass, height and leg length) and the extreme range of step lengths (0.24–1 m), a remarkably small

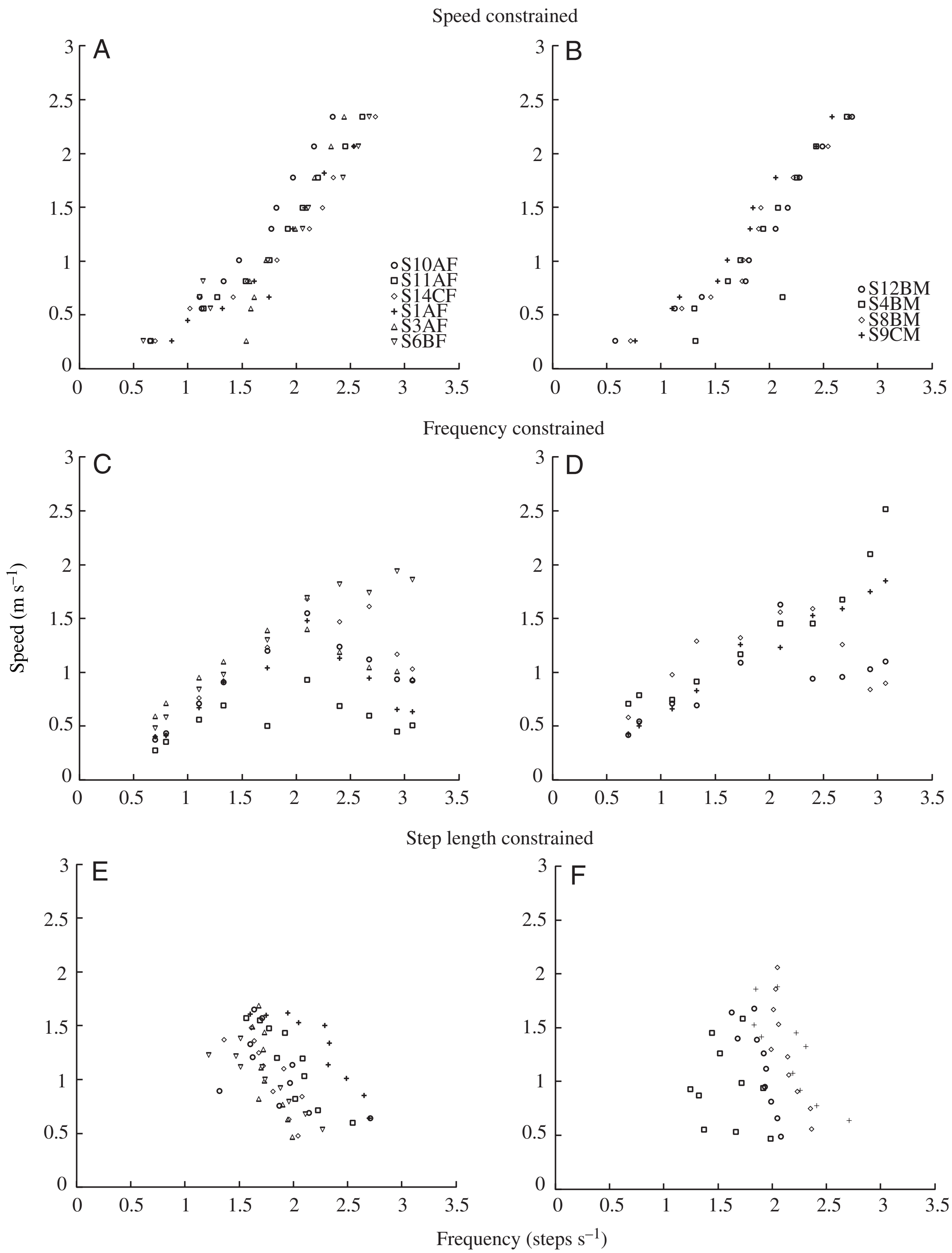

Fig. 6. Speed–frequency data for individual subjects as they responded to the applied constraints. Subject labels are consistent between plots within each gender and match the labels indicated in the insets of A and B. These data are those plotted in Fig. 5, but have been reproduced in this format to allow tracking and comparison of individual responses.

range of frequencies were employed. This small range of frequencies bracketed the frequency used at the preferred walking speed, suggesting the natural frequency of the swing leg was a major determinant of this aspect of the step under constrained step lengths. In spite of the limited frequencies used, a range was observed. This simply indicates that the motion of the swing leg is not the only determinant of walking cost, and other rate-related costs also need to be considered.

Such factors as negative work and collision losses in the contact limb (Donelan et al., 2002; Ruina et al., 2005) would contribute to locomotion cost in step length constrained walking, just as they do for speed or frequency constraints. The main difference in step length constrained walking is that, within the assigned task, motion of the swing limb becomes a more determining factor of the overall cost function. Thus, even the apparently odd relationship between speed and frequency in step length constrained walking can, on closer inspection, be seen as a logical (natural) response to the conditions imposed.

Interpreting the results

If it can be assumed that the walking parameters selected by the subjects under each of these constraint conditions are not an artifact, how well does the constrained optimization hypothesis, using metabolic cost per distance traveled as the objective function, explain the selection of gait parameters?

(i) Speed constraint

The behavioral data generated in speed constrained conditions matches that observed in numerous previous studies under these same circumstances (Minetti et al., 1994; Nilsson et al., 1985) and fits well documented cost data (Atzler and Herbst, 1927; Margaria, 1938; Ralston, 1958). Subjects walking on a treadmill are expected to select this speed–frequency relation, and this is attributed to the step frequency–step length combinations providing cost minimization for each speed (Kuo, 2001). As such, these data provide no surprises other than their relationship to the other constraints employed in the present study. If walking parameters are selected in normal, speed constrained conditions in the same manner as in notable unusual circumstances like frequency and step length constraints, then the walking relations generated for the alternative constraints may be viewed simply as normal walking responses under those constraints. The functional plasticity displayed in these other constraints suit the imposed circumstances, just as speed constrained walking does under its constraint condition (the required speed).

(ii) Frequency constraint

The data generated for frequency constrained walking are simultaneously the most complex, the least closely related to the prediction and potentially the most telling regarding the constrained optimization hypothesis. At the lower step frequencies (below 1.5 steps s^{-1}) the mean speed selected for each frequency was systematically above the optimization (by approximately 1 S.D.). This is in a region of the cost surface where the contours predict a sharp optimum (i.e. the contours are highly curved, indicating a substantial cost for deviating from the optimum). At these lower frequencies the metabolic cost rate is relatively low, even if the cost/distance is higher. Since the subjects were not required to walk for extended periods, it is possible that they selected higher walking speeds because they were not familiar with walking at such low speeds, i.e. basically out of impatience with the experimental protocol, and the deviation was allowed by the low cost rate even though walking at those speeds was relatively inefficient. It is also possible that at such low speeds features of the swing and stance period were dissociated, giving a different optimization solution. Changes in relative stance and swing period were not measured in this study but may be useful to consider in future studies. The predicted cost minimization should hold, however, unless differences existed between frequency constrained over-ground walking (where gait selection was measured) and the speed–frequency–step length constrained treadmill walking in which walking cost was assessed. Alternatively, the deviation from the model's expectation may indicate the influence of optimization criteria not considered in the current model. For instance, if cost minimization at these or other speeds was influenced in some part by a penalty for going slowly or by such considerations as cost/step (rather than the cost/distance considered here), then the observed behavior might fit an alternative model. Evaluation of such alternatives will likely require a sophisticated set of experiments involving isolation of specific variables influencing the subject's perception of the task at hand. Such information is not available from the current study. The systematic deviation from the model closely follows the slope of the optimization in this region of the curve, which may suggest some influence on its determination by the cost profile. Full explanation of this aspect of the model and these differences between prediction and selected behavior will require further evaluation, possibly with substantially longer duration walking tasks and/or alternative constraint regimes.

At a constrained step frequency of 2.1 steps s^{-1} all subjects but one walked at a speed exceeding preferred walking speed (1.1 m s^{-1}, Ralston, 1958; Sun et al., 1996); the exception consistently walked at substantially lower speeds than any other subject (Fig. 6D). At this step frequency the cost contours indicate a single optimum speed that lies very close to the mean selected by the subject group (Fig. 5B). Beyond this frequency, however, two distinct optima are indicated by the cost contours, one at increasing speeds and another centered slightly below preferred walking speed. For all frequencies above 2.1 steps s^{-1} the subject pool spontaneously divided into two groups, one composed of four subjects that appeared to choose the higher speed optimum and a second composed of the remaining subjects that walked at decreasing speeds as frequency increased. The mean behavior of each of the groups matched well with one or the other optimization opportunity available in this region, as indicated by regression of each subgroup (Fig. 5B). This is compelling evidence that the selection of walking parameters is indeed strongly influenced by constrained optimization of the metabolic cost. The match between the selected behavior and the multiple optimizations in this region is even more striking when it is observed that, despite the high variability in these data, no speeds intermediate between the two optima were selected, i.e. the scatter in these data is centered on each of the two optima rather than evenly distributed over the entire range.

(iii) Step length constraint

In step length constrained walking the optimization prediction itself was not well determined due to the congruency of cost contours and the slope of the step length constraint. This feature can be seen by comparing the slope of the speed–frequency data for each step length (where speed and step frequency covary at a slope determined by the step length) and the slope of the cost contour in the region of the data. The cost contours and step length determined slopes are nearly the same throughout the range of step lengths used (Fig. 5C). As a result of this congruence there is little or no cost consequence for a wide range of speed–frequency combinations and the ±1% and ±5% optimization solutions meander back and forth across the data. These drifts in optimum position likely indicate subtle variations in the cost profile that remain because a minimal level of smoothing was used to generate the cost surface from the original discrete point data, or might result from subtle artifacts produced by the complex Fourier surface modeling routine. It is likely that such minor fluctuations in cost are not meaningful, particularly for pooled data such as these. This remains to be determined by future studies. The ±10% and ±15% ranges describe a broad swath of options near the optimum that include a large proportion of those selected by the subjects. This shows a general insensitivity of the speed–cost relation at any given step length, though lower overall costs are associated with step lengths in the normal range. Perhaps this cost insensitivity is indicative of a useful design feature allowing for flexibility in step length selection and the negotiation of obstacles that would routinely obstruct the path of a walker in natural habitats. As discussed above, some aspects of the cost congruence and behavior selection likely derive from the pendular action of the swing limb, but a complete understanding of these features will depend on a full exploration of the relation between walking dynamics and metabolic cost.

Implications

Of interest in the context of the general applicability of the constrained optimization hypothesis is the indication from the current data that the same explanation, constrained optimization of locomotion cost, can predict general features of the self-selected response to both commonly (speed) and uncommonly (frequency and step length) imposed walking conditions. In this case, the control of a single feature of the walking gait (either speed, step frequency or step length) appears adequate to elicit the myriad of other coordination factors (nerve firing, muscle activation and force stimulation, feedback integration, etc.) that ultimately result in coordinated (and effective) walking. Such responsiveness suggests the coordinating system has both substantial plasticity and is able to spontaneously respond to the integrated physical and physiological conditions with which it is confronted. At this point not enough information is available to determine how this is accomplished in human walking, but the results of experiments like those described here indicate that new opportunities are available to explore the details of the process and the mechanisms responsible.

Although a reasonable correspondence occurs between the observed behavior under each of the three walking constraints and the predicted behavior using metabolic cost per distance (J kg^{-1} m^{-1}), there are several instances where the predictions are not as close as one might expect if cost/distance is indeed the determining objective function. Notable differences between cost minimization strategy and gait parameter selection suggest that factors other than global cost of travel also influence the control of human walking. Such consistent differences parallel previously identified cost anomalies in gait such as the preference to change gait between walking and running at speeds in which apparent cost is not minimized (Hreljac, 1993; Thorestensson and Roberthson, 1987; Tseh et al., 2002). It is likely that the observed differences indicate that cost/distance is not the only factor responsible for the behavior selected. The complete objective function may well prove to be more complex, involving partial contributions from a number of considerations. Such a function (F) could take the form:

$$F = C_0(\text{cost/dist}) + C_1(\text{cost/time}) + C_2(\text{cost/step}) + C_3(v) + \ldots C_i(F_i)\ ,$$

where C_0–C_i are proportional constants indicating the influence of each contributing function. Such contributing functions could range from the currently considered cost/distance through cost/time, cost/step, velocity (v) and other currently unidentified functions (F_i).

From the present study it is apparent that the cost/distance function has a substantial influence on the cost surface that makes up the landscape of the objective function, but each additional component could contribute a modifying influence. By providing a means to experimentally explore the process by which the human locomotion system selects the general features of gait, and by implication also the underlying specifics, constrained optimization provides a strategy for evaluating the factors that determine the control of walking in humans. The current study provides only a superficial examination of the potential for this line of examination. Constrained optimization implies that gait selection parameters should be predictable on an individual basis, rather than the general group characteristics considered here. The present work has demonstrated that taking the analysis to this level will require consideration of numerous potentially confounding factors such as individual motivation and experience with the task. The constrained optimization perspective, however, provides a new strategy for exploring this aspect of walking control and determining the role of these fundamental factors.

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